# PRACTICAL MECHANICS.

# THE

# APPRENTICE,

OR

# FIRST BOOK

FOR

# MECHANICS, MACHINISTS,

AND

# ENGINEERS.

BY

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# PREFACE.

THERE being no Elementary treatise on Practical Mechanics in the English language, the present work is designed to supply the deficiency. It may be asked how can this be, while there exist so many elementary treatises on the theory and application of Mechanics, or why not translate, or copy the great French works, as the German and other nations do. To understand the elementary treatises on the theory and application of Mechanics employed as class books in our schools and colleges, requires considerable mathematical skill, and after all, the elementary know ledge acquired is of little value to the practical man. On the general theory of Mechanics, there are also many popular works clear of mathematical formula and serviceable to the general reader, but of no practical value. French elementary writers on practical Mechanics, take for granted that they are writing for those who understand the theory of Mechanics, and when these works are copied or translated, they are far from being elementary.

As practical mechanics is based upon what is termed the principle of work, to be able to distinguish with ease, *units of work* from other units, would greatly facilitate the acquisition of this branch of knowledge; I have, therefore, introduced a new and simple notation to effect this object, which French writers and their copyists have neglected.

The *unit of work* is equal to the labor required to raise

a pound weight through the space of one foot, against the direct action of the force of gravity. If a man takes a pound weight in his hand, and raise it one foot in the direction of a plumb-line, he will perform a unit of work.

1 lb. raised 24 feet high = 24 units of work ;
2 lbs. raised 12 feet high = 24 units of work ;
3 lbs. raised 8 feet high = 24 units of work ;
4 lbs. raised 6 feet high = 24 units of work ;
6 lbs. raised 4 feet high = 24 units of work ;
8 lbs. raised 3 feet high = 24 units of work ;
12 lbs. raised 2 feet high = 24 units of work ;
24 lbs. raised 1 foot high = 24 units of work.

To distinguish units of work from other units, I simply write or set down the numbers that represent them, inclining to the left, instead of the ordinary way, thus :

**24** represents 24 units of work ;

$$8 \times 3 = 24 ;$$

that is the same as saying, 8 lbs. raised 3 feet high is equal to 24 units of work.

**24**′ ; represents 24 units of work done in a minute.

**24**″ ; represents 24 units of work done in a second.

$$33000' = 1^{\text{h.}} ;$$

represents 33,000 units of work done in a minute, equal one horse power.

I have been constantly asked the question, by those whose design it was to become machinists and engineers, "What is the best elementary work on PRACTICAL MECHANICS ?" ; instead of answering, "there is no such work in the English language," I can now say, buy "The APPRENTICE," the title given to this work, to intimate that it is designed for persons who wish to acquire a knowledge of Practical Mechanics.

OLIVER BYRNE.

# PRACTICAL MECHANICS.

## CHAPTER I.

*On the unit of work with, and without reference to tne unit of time.*

WORK is measured by a unit, like length, weight, time, &c. To raise one pound a foot high against the earth's gravity is a unit of work; it is clear, then, to raise 5 lbs. a foot high is to perform 5 units of work, and to raise 5 lbs. four feet high equal 20 units of work. Since resistance and pressure of every kind may be expressed in pounds, it follows that the unit here described may be made to measure every kind of work. It will be presently shown that a unit of work is performed whenever one pound pressure is exerted through a space of one foot, no matter in what direction the space may lie.

In developing this subject it is necessary to distinguish units of work from other units; this I propose to do by writing or setting down the numbers that represent them *inclining to the left* instead of in the ordinary way, thus,

**25** represents 25 units of work.

**25**′ will represent 25 units of work done in a minute.

**73**″ will represent 73 units of work done in a second.

Sometimes the minute is taken for the unit of time, and 33 minutes is marked 33′; in other cases, the second is taken as the unit of time, and 45 seconds is marked 45″.

45″ represents 45 seconds;

but **45**″ units of work done in a second of time.

a′ represents a minutes, but **a**′ represents **a** units of work done in one minute.

EXAMPLES.

*Question* 1. Required the units of work expended in raising a weight of 50 lbs. to the height of 31 feet.

Units of work in raising 1 lb. 31 feet = **31**.

$$\therefore 50 \times \mathbf{31} = \mathbf{1550}.$$

the required units of work.

*Ques.* 2. The ram of a pile-driving engine weighs half a ton, and has a fall of 17 feet; how many units of work are performed in raising this ram?

Half a ton = 1120 lbs.

Units of work in raising 1 lb. 17 feet = **17**.

$$\therefore 1120 \times \mathbf{17} = \mathbf{19040},$$

the units of work required to raise 1120 lbs. to a height of 17 feet.

*Ques.* 3. How many units of work are required to raise 7 cwt. of coal from a pit whose depth = 13 fathoms?

$$7 \times 112 = 784 \text{ lbs.}$$

$$13 \times 6 = 78 \text{ feet, in 13 fathoms.}$$

Hence the work consists in raising 784 lbs. to the height of 78 feet.

$$\therefore 784 \times 78 = \mathbf{61152}.$$

*Ques.* 4. If the weight of a man be 183 lbs., and if he ascends a perpendicular height of 20 feet, he does work in raising himself, what are the number of units?

In this operation the man raises the weight of his own body,

$$\therefore 183 \times 20 = \mathbf{3660}.$$

amount of work.

If this man were to descend in a bucket, it is clear, he would perform the same work upon a counterpoise weight when he has descended 20 feet.

*Ques.* 5. How many units of work will be required to pump 8000 cubic feet of water from a mine whose depth = 500 fathoms?

A cubic foot of water weighs 62.5 lbs.

$$8000 \times 62.5 = 500{,}000 \text{ lbs.}$$

$$500 \text{ fathoms} = 3000 \text{ feet.}$$

Consequently, the work is to raise 500,000 lbs. a perpendicular height of 3000 feet.

$$\text{Work} = 500{,}000 \times 3000 = \mathbf{1500000000}.$$

*Ques.* 6. A horse draws 150 lbs. out of a well, by means of a rope going over a fixed pulley, moving at a rate $2\frac{1}{2}$ miles an hour, how many units of work does this horse perform a minute, the friction being neglected?

$$\frac{2\frac{1}{2} \times 5280}{60} = 220 \text{ feet passed over per minute.}$$

$$\therefore \text{ Work per minute} = 150 \times 220 = \mathbf{33000}'.$$

$$\text{Work per second} = \mathbf{500}''.$$

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## THE UNIT OF WORK REFERRED TO A UNIT OF TIME.

A unit of work, or **1**, represents 1 lb. raised 1 foot.

A unit of work in a minute, or **1**′, represents 1 lb. raised a foot high in a minute.

A unit of work in a second, or **1**″, represents 1 lb. raised a foot high in a second.

It has been assumed that a horse is capable of raising 33000 lbs. a foot high in a minute, or to perform **33000** units of work in a minute,

$$\text{Hence a horse power} = \mathbf{33000}' = \mathbf{1}^{\text{h.}}$$

Whether this power is greater or less than the power of a horse it matters little, while it is a power so well defined.

To raise 10,000 lbs. a foot high in a minute, would be a more convenient unit to measure by.

## EXAMPLES.

*Question* 1. How many horse power would it take to raise 3 cwt. of coal per minute from a pit whose depth = 110 fathoms?

$$\text{Depth} = 110 \times 6 = 660 \text{ feet.}$$

$$3 \text{ cwt.} = 112 \times 3 = 336 \text{ lbs.}$$

$$660 \times 336 = \mathbf{221760'}.$$

Since a horse power = $\mathbf{33000'}$

$$\frac{\mathbf{221760'}}{\mathbf{33000'}} = \mathbf{6\ ^{h}.72}, \text{ the required power.}$$

*Ques.* 2. How many horse power is required to raise 2200 cubic feet of water an hour from a mine whose depth = 63 fathoms?

$$137500 = 2200 \times 62.5 = \text{weight of water in lbs.}$$

$$63 \times 6 = 378 \text{ feet, depth of mine.}$$

$$\frac{137500 \times 378}{60} = \mathbf{866250'}.$$

$$\therefore \quad \frac{866250}{33000} = \mathbf{26\tfrac{1}{4}\ ^{h.}}$$

The proposed notation, must be borne in mind, $\mathbf{866250'}$ signifies 866250 lbs., raised one foot high in a minute, and $\mathbf{26\frac{1}{4}\ ^{h.}}$, represents $26\frac{1}{4}$ horse power, or $26\frac{1}{4}$ times 33000 lbs. raised 1 foot high in a minute.

*Ques.* 3. A winding engine is moved by 4 horses, what weight of coal will be raised per hour from a pit whose depth = 200 feet?

Work of the 4 horses in an hour,

$$= 4 \times 33000 \times 60 = \mathbf{7920000}.$$

Work in raising 1 lb. of coal 200 feet = $\mathbf{200}$.

Since it takes 200 units of work to raise a pound of coals,

$$\frac{7920000}{200} = 39600 \text{ lbs.}$$

Consequently four horses, will raise 39600 lbs. of coal or of any thing else, a height of 200 feet in an hour.

*Ques.* 4. In what time will an engine of 10 horse power, raise 5 tons of material from the depth of 132 feet?

5 tons = 11200 lbs.

Work of the engine per minute,

$$= \mathbf{330000}' = 33000 \times 10.$$

Work of raising 5 tons,

$$= 11200 \times 132 = \mathbf{1478400}.$$

Because the engine performs 330000 units of work a minute,

$$\therefore \quad \frac{1478400}{330000} = 4.48 \text{ minutes.}$$

*Ques.* 5. How many cubic feet of water will an engine of 36 horse power (**36** $^{h.}$) raise in an hour from a mine whose depth is 40 fathoms?

$$40 \times 6 = 240 \text{ feet.}$$

Work in an hour,

$$= 36 \times 33000 \times 60 = \mathbf{71280000}.$$

Work to raise a cubic foot of water 240 feet,

$$= 240 \times 62.5 = \mathbf{15000}.$$

$$\frac{\mathbf{71280000}}{\mathbf{15000}} = 4752 \text{ cubic feet.}$$

*Ques.* 6. From what depth will an engine of 22 horse power, raise 13 tons of coal in an hour?

Work done by the engine in an hour,

$$22 \times 33000 = \mathbf{726000}.$$

13 tons = 29120 lbs.

$$\frac{\mathbf{726000}}{29120} = 24.9 \text{ feet.}$$

*Ques.* 7. An engine is observed to raise 7 tons of material per hour, from a mine whose depth is 85 fathoms; required the horse power of the engine, supposing $\frac{1}{6}$ of its work to be lost in transmission?

$$\text{Work per min.} = \frac{2240 \times 7 \times 85 \times 6}{60} = \mathbf{133280}.$$

Since $\frac{5}{6}$ of the work of the engine only go to raise the material,

$$\therefore \quad \tfrac{5}{6} \text{ of } \mathbf{33000'} = \mathbf{27500'},$$

the units of useful work of one horse power a minute.

$$\frac{\mathbf{133280'}}{\mathbf{27500'}} = \mathbf{4^{h}.4829}.$$

*Ques.* 8. Required the horse power of an engine that would supply the city of Brooklyn with water, working 12 hours a day, the water to be raised to a height of 50 feet; the number of inhabitants = 130,000, and each person to use 5 gallons of water a day.

The standard gallon of the United States weighs $8\frac{1}{3}$ lbs., nearly.

$$\frac{130000 \times 5 \times 8\frac{1}{3}}{12 \times 60} = \frac{8125 \times 25 \text{ lbs.}}{27}$$

the pounds of water to be pumped in a minute.

$$\therefore \quad \frac{8125 \times 25}{27} \times 50 = \mathbf{376157'}$$

$$\therefore \quad \frac{\mathbf{376157'}}{\mathbf{33000'}} = \mathbf{11^{h}.4}.$$

The horse power required?

*Ques.* 9. What is the horse power of an engine that pumps from three different levels, whose depths are 40, 70, and 90 fathoms respectively; from the first 20 cubic feet of water are raised per minute, from the second, 10 cubic feet, and from the third, 35 feet, allowing $\frac{1}{3}$ the work of the engine to be destroyed by useless resistance?

$$\mathbf{7^{h}.} = \mathbf{33000'}.$$

$$\tfrac{2}{3} \text{ of } \mathbf{33000'} = \mathbf{22000'},$$

the effective power of the engine.

1st level, work $= 62.5 \times 20 \times 240 =$ **300000'.**
2nd " " $= 62.5 \times 10 \times 420 =$ **262500'.**
3rd " " $= 62.5 \times 35 \times 540 =$ **1181250'.**
**1743750'.**

$$\therefore \quad \frac{\mathbf{1743750}}{\mathbf{22000'}} = \mathbf{79^{h}.26}.$$

*Ques.* 10. There were 6000 cubic feet of water in a mine whose depth is 60 fathoms, when an engine of 50 horse power began to work the pump; the engine continued to work 5 hours before the mine was cleared of the water; required the number of cubic feet of water which had ran into the mine per hour, supposing $\frac{1}{4}$ of the work of the engine to be lost by transmission?

$$\begin{array}{r} 4)\mathbf{33000'} \\ \mathbf{8250} \\ \hline \mathbf{24750'} \end{array}$$ effective power of the

engine a minute. It must not be forgotten that **24750'** written in the contrary direction, signifies 24750 units of work in a minute or 24750 lbs., raised one foot high in a minute; so that writing down the numbers in this peculiar manner saves much written explanation.

$$\mathbf{24750'} \times 50 \times 5 \times 60 = \mathbf{372250000},$$

effective work of the engine in 5 hours.

$$62.5 \times 60 \times 6 = \mathbf{2250},$$

work in raising one cubic foot of water to a height of 60 fathoms = 360 feet.

$$\frac{\mathbf{372250000}}{\mathbf{22500}} = 16544.44,$$

cubic feet of water pumped in 5 hours.

$$\begin{array}{r} 16544.44 \\ 6000. \\ \hline 5)\ 10544.44 \\ \hline 2108.88 \end{array}$$ Water run in during 5 hours.

Cubic feet run in in one hour.

*Ques.* 11. A forge hammer weigns 300 lbs., makes 100 lifts a miuute, the perpendicular height of each lift = 2 feet; what is the horse power of the engine that gives power to 20 such hammers?

Work of each lift = $300 \times 2 \times 20 = \mathbf{12000}$.

Work in 100 lifts, that is, in one minute,

$$\mathbf{12000} \times 100 = \mathbf{1200000}.$$

$$\frac{\mathbf{1200000'}}{\mathbf{33000'}} = \mathbf{36^{h}.36}.$$

*Ques.* 12. An engine of 10 horse power ($\mathbf{10}^{h.}$), raises 4000 lbs. of coal from a pit 1200 feet deep in an hour, and also gives motion to a hammer which makes fifty lifts in a minute, each lift having a perpendicular height of 4 feet, what is the weight of the hammer?

Work done by the engine in one minute,

$$= \mathbf{33000} \times 10 = \mathbf{330000}'.$$

The units of work performed, in raising the coals, per minute =

$$\frac{1200 \times 4000}{60} = \mathbf{80000}'.$$

Work engaged in raising the hammer per minute,

$$= \mathbf{330000}' - \mathbf{80000}' = \mathbf{250000}'.$$

Work per minute, in raising one lb. of hammer 4 feet high, 50 times a minute, =

$$1 \times 4 \times 50 = \mathbf{200}'.$$

$$\frac{\mathbf{250000}'}{\mathbf{200}'} = 1250 \text{ lbs},$$

the weight of the hammer required.

## CHAPTER II.

### *On the work of living agents.*

THE laboring force of animals, varies very much with the way in which their muscular strength is exerted; and also, with the rate at which they labor.

The following little table shows the greatest amount of effective work that a laboring man can perform under the different modes in which he may exert his muscular power.

WORK DONE BY A MAN PER MINUTE, WHEN HE WORKS 8 HOURS A DAY.

A man in raising his own body,.....**4250**.
A man in working at a treadmill ..... ...**3900**'.

A man drawing, or pushing horizontally,.....**3120′**.
A man pushing or pulling vertically,.....**2380′**.
A man turning a handle, .....**2600′**.
A man working with arms and legs, as rowing,**4000′**.

UNITS OF WORK DONE BY A MAN PER MINUTE, WHEN HE WORKS **6** HOURS A DAY.

A man in raising material with a pulley.....**1560′**.
A man in raising material with the hands,.....**1470′**,
A man in raising material on back, returning empty,.**1126′**.

UNITS OF WORK DONE BY A MAN PER MINUTE, LABORING **10** HOURS A DAY.

Raising materials with a wheel-barrow on ramps....**720′**.
Throwing earth to the height of 5 feet.....**470′**.

UNITS OF USEFUL WORK DONE BY A MAN IN A MINUTE, RAISING WATER BY DIFFERENT ENGINES, WORKING **8** HOURS A DAY.

With a windlass from deep wells,.....**2560′**.
With an upright chain pump, .....**1730′**.
With a tread-mill, .....**3176′**.
With a Chinese wheel, .....**2167′**.
With an Archimedian screw, .....**1505′**.
Raising water from a well, with rope and pail,....**1054′**.

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## WORK OF BEASTS.

THE imaginary horse by which the power of steam engines are measured, is supposed to do more work than the general run of horses are able to perform. The work of a horse of average strength is about 22000 pounds raised one foot high in a minute, which according to my notation, is expressed thus :—

**22000′**.

A mule will perform $\frac{2}{3}$ the work of a horse, = **14666$\frac{2}{3}$′**.

An ass will perform about $\frac{1}{6}$ the work of a horse $= \mathbf{4400'}$.

In a common pumping engine a horse of average strength will do only 17550 useful units of work in a minute.

From $\mathbf{22000'}$
Take $\mathbf{17550'}$
$\mathbf{4450'}$, lost in this case by friction and useless resistances.

EXAMPLES.

*Question* 1. How many cubic feet of clay, weighing 100 lbs. per cubic foot, will 20 men throw 5 feet high in a day of 10 hours long?

From the foregoing table the number $\mathbf{470'}$ is found.

Work in a day,

$$= \mathbf{470'} \times 60 \times 10 \times 20 = \mathbf{5640000'}.$$

Work in raising one cubic foot,

$$= 100 \times 5 = \mathbf{500'}.$$

Consequently the number of cubic feet raised in one day by 20 men,

$$= \frac{\mathbf{5640000}}{\mathbf{500}} = 11280.$$

*Ques.* 2. How many bricks will a man raise in a day six hours long to a height of 30 feet, supposing the weight of a cubic = 125 lbs., and 17 bricks to form a cubic foot?

For a man working in this way, the foregoing tables give $\mathbf{1126'}$.

$$\therefore \quad \mathbf{1126'} \times 60 \times 6 = \mathbf{405360},$$

work done by a man in 6 hours.

$$125 \times 30 = \mathbf{3750},$$

work done in raising a cubic foot 30 feet high.

$$\therefore \quad \frac{\mathbf{405360}}{\mathbf{3750}} = 108.096,$$

cubic feet raised in a day of 6 hours.

Number of bricks $= 108.096 \times 17 = 1838$.

*Ques.* 3. How many cubic feet of water will a laborer working with a bucket and rope, raise from a well whose depth is 16 feet?

In the table will be found **1054′**, for a man working in this manner.

$$\mathbf{1054'} \times 60 \times 8 = \mathbf{505920},$$

work done in 8 hours.

The work done in raising a cubic foot of water 16 feet high,

$$= 6.25 \times 16 = \mathbf{1000}.$$

Hence the cubic feet raised in eight hours by a man,

$$= \frac{\mathbf{505920}}{\mathbf{1000}} = 505.92.$$

*Ques.* 4. What weight is a man, working at a tread-mill, able to raise in a day of 8 hours, from a depth of 110 feet?

Tabular number $= \mathbf{1730'}$.

$$\mathbf{1730'} \times 60 \times 8 = \mathbf{830400},$$

work done in a day.

The work required to raise one cwt. or 112 lbs. a height of 110 feet will be,

$$112 \times 110 = \mathbf{1232};$$

$$\therefore \frac{\mathbf{830400}}{\mathbf{1232}} = 674 \text{ cwt.},$$

the weight that may be raised by a man working at a tread-mill.

*Ques.* 5. How many tons of coal would a man raise, working with a wheel and axle, from a pit whose depth is 20 feet, taking for granted, according to the tabulated statement, that a man so circumstanced, can perform 2600 units of work in a minute?

The work done in one day of 8 hours,

$$= \mathbf{2600'} \times 60 \times 8 = \mathbf{1248000}.$$

The work to raise one ton,

$$= 2240 \times 20 = \mathbf{44800};$$

$$\therefore \frac{\mathbf{1248000}}{\mathbf{44800}} = 27.86, \text{ tons}$$

raised in a day of eight bours.

*Ques.* 6. The ram of a pile engine weighs 7 cwt., and has a fall of 23 feet, how many strokes a day, will four men give, working at a wheel and axle?

Work done in one day of eight hours,

$$= \mathbf{2600'} \times 60 \times 8 \times 4 = \mathbf{4992000}.$$

The work of one stroke,

$$= 112 \times 7 \times 23 = \mathbf{1832};$$

$$\therefore \frac{\mathbf{4992000}}{\mathbf{1832}} = 277 \text{ strokes nearly.}$$

The force with which animal pull decreases with their speed; the relation between the traction and speed of a horse, is expressed with considerable accuracy by the following formula,

$$t = 250 - 41\tfrac{2}{3}\, r;$$

in which $t$ = the traction in lbs. and $r$ = the rate in miles per hour.

Hence if the speed be 3 miles an hour, the traction will be 125 lbs., for

$$250 - 41\tfrac{2}{3} \times 3 = 125 \text{ lbs.}$$

If the speed be one mile an hour, the traction will be $= 208\frac{1}{3}$ lbs., for

$$250 - 41\tfrac{2}{3} \times 1 = 208\tfrac{1}{3} \text{ lbs.}$$

From this formula, it is easily shown that a horse will do the greatest amount of work when he travels at the rate of 3 miles an hour, in fact

$$\left\{250 - 41\tfrac{2}{3}\, r\right\} \times r$$

becomes a maximum when $r = 3$.

If a body move slowly along a horizontal plane, the resistance to be overcome is called friction. Experiment has determined that this resistance on a given surface, is a fractional part of the weight of the body moved, and it has also been found that any change in the rate of the motion of the body does not affect the resistance due to friction; nor is the amount of friction altered by the extent of the rubbing surfaces.

When a wagon or a cart is drawn along a good common road, the resistance of friction is about $\frac{1}{30}$ of the whole load, so that a horse in order to draw 3120 lbs. along a road, must pull with the force of 104 lbs., for

$$\frac{3120}{30} = 104 \text{ lbs.}$$

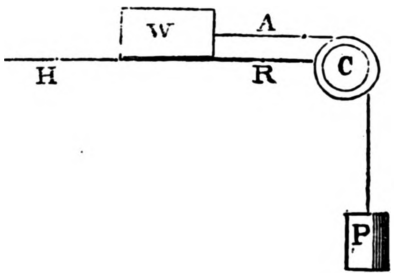
A horse with this traction, according to the foregoing formula, moves at the rate of about $3\frac{1}{2}$ miles an hour, for

$$t = 250 - 41\tfrac{2}{3}\, r,$$

$$\text{becomes } 104 = 250 - 41\tfrac{2}{3}\, r,$$

$$\therefore \quad r = 3\tfrac{1}{2}.$$

A carriage on a Rail road, only requires a pressure of about $\frac{1}{300}$ part of the moving weight to give it motion, or from 4 to 8 lbs. a ton. The fractions $\frac{1}{30}$ for common roads, and $\frac{1}{300}$ for rail-roads are called the coefficients of friction, as these coefficients become smaller, the rubbing surfaces become smoother.




Let W be a weight drawn on a horizontal plane H R, by means of a weight P, attached to a cord A, going over a fixed pulley C; then the weight P just necessary to draw or move W, along the plane, will be equal to the resistance of friction. In the case of a Rail-road, if W = 150 tons, P will be = 900 lbs., when the coefficient of friction = $\frac{3}{120}$ or 6 lbs. a ton.

It is very evident that whatever distance P descends, the weight W, will be drawn along the plane H R, the same distance; hence the units of work done in moving W, will be the weight of P in pounds multiplied by the distance in feet through which it descends, or the resistance of friction in pounds, multiplied by the space in feet over which W, is moved.

The work of every machine is consumed by the work done, or by the useful work, together with the useless work, or the work destroyed by the friction of the parts of the machine. I will here explain one of the most beautiful laws of motion;—when the work applied exceeds the work consumed, the redundant work goes to increase the speed of the parts of the machine, and at the same time like the fly-wheel, acts as a reservoir of work. *This acceleration goes on increasing until the work of the resistances + the useful work = the work applied; and then the motion of the machine becomes uniform.*

For example, in a Rail-road engine and train, at first the work of the engine exceeds the work of the resistances, and hence the speed of the engine goes on increasing; but, as the speed increases, the work of the resistances also increases, so that ultimately the engine attains a nearly uniform motion, which is called the greatest or maximum speed, and then the work destroyed by the resistances, will be exactly equal to the work applied by the moving power.

*Ques.* 7. Required the effective horse power of a locomotive engine, which moves at a steady speed of 23 miles an hour upon a level rail, the weight of the train being 100 tons, and the friction 5 lbs. a ton?

Put $x$ = the required horse power.

The work of the engine per minute,

$$= x\,\mathbf{33000'};$$

The resistance of friction,

$$= 5 \times 100 = 500 \text{ lbs.}$$

The distance moved per minute,

$$= \frac{23 \times 5280}{60} = 2024 \text{ feet};$$

Work to overcome the friction,

$$= 2024 \times 500 = \mathbf{1012000'}.$$

But as the speed of the train is uniform, the work of the resistances will be equal to the effective work of the engine;

$$\therefore\ x \times \mathbf{33000'} = \mathbf{1012000'}$$

$$\therefore\ x = \frac{\mathbf{1012000'}}{\mathbf{33000'}} = \mathbf{30^{h}.7}.$$

*Ques.* 8. What is the rate in miles per hour, of a train of 80 tons will be drawn by an engine of $\mathbf{70^{h.}}$; the friction 8 lbs. to the ton.

Call $x$ the uniform speed in miles per hour;

Work used in moving the train $x$ miles,

$$= 80 \times 8 \times 5280\,x;$$ this

is the work done by the engine in an hour. But the work done by the engine in an hour, will also be expressed by

$$\mathbf{33000'} \times 70 \times 60;$$

$$\therefore\ \mathbf{33000'} \times 70 \times 60 = 80 \times 8 \times 5280\,x;$$

$$\therefore\ x = \frac{\mathbf{33000'} \times 70 \times 60}{80 \times 8 \times 5280} = 41.02 \text{ miles.}$$

*Ques.* 9. An engine of $\mathbf{48^{h.}}$, moves with a maximum speed of 33 miles an hour, on a level rail; required the gross load of the train, friction 6 lbs. per ton?

Let $x$ be the gross weight of the train in tons, then, the work consumed per hour in moving the train,

$$= x \times \mathbf{6} \times \mathbf{33} \times \mathbf{5280}.$$

Work of the engine per hour,

$$= \mathbf{48} \times \mathbf{33000}' \times \mathbf{60}.$$

When the speed is uniform or at its maximum,

$$x \times \mathbf{6} \times \mathbf{33} \times \mathbf{5280} = \mathbf{48} \times \mathbf{33000} \times \mathbf{60};$$

$$\therefore \quad x = \frac{48 \times 33000 \times 60}{6 \times 30 \times 5280} = 90\tfrac{10}{11} \text{ tons.}$$

*Ques.* 10. In what time will an engine of 66 horse power moving a train of 200 tons, complete a journey of 100 miles, friction 5 lbs. per ton ; rails horizontal ?

Work expanded in moving the train 100 miles,

$$= 100 \times 5280 \times 200 \times 5 = \mathbf{528000000}.$$

Work of the engine per hour,

$$= \mathbf{33000}' \times 66 \times 60 \times \mathbf{130680000}$$

$$\frac{\mathbf{528000000}}{\mathbf{130680000}} = 4.04 \text{ hours.}$$

*Ques.* 11. What work per minute will a horse perform when traveling at the rate of $2\frac{1}{2}$ miles an hour ?

I have before shown that the traction of a horse moving at the rate of $2\frac{1}{2}$ miles an hour,

$$= 250 - 41\tfrac{2}{3} \times 2\tfrac{1}{2} = 145\tfrac{5}{6} \text{ lbs.}$$

$2\frac{1}{2}$ miles $= 13200$ feet per minute,

$$= \frac{13200}{60} = 220 \text{ feet.}$$

Hence the work of this horse per minute,

$$= 145\tfrac{5}{6} \times 220 = \mathbf{32083}\tfrac{1}{3}'.$$

*Ques.* 12. What load will a horse draw, traveling at 3 miles an hour upon a plank road, whose friction is $\frac{1}{100}$ of the whole load, the road being horizontal ?

The traction at this speed is 125 lbs.,

$$\text{for } 250 - 41\tfrac{2}{3} \times 3 = 125.$$

The gross load must be 100 times this weight,

$$= 125 \times 100 = 12500 \text{ lbs.}$$

*Ques.* 13. Suppose a horse to be able to perform 33000 units of work in a minute on a horizontal road, whose friction is $\frac{1}{20}$ of the whole road; required the load and rate per hour?

$$\text{Traction} = (250 - 41\tfrac{2}{3}\, r) \text{ pounds.}$$

Space passed over in a minute,

$$= \frac{5280\, r}{60} = 88\, r \text{ feet.}$$

Then $(250 - 41\frac{2}{3}\, r)\, 88\, r =$ the units of work performed by the horse in a minute $= \mathbf{33000}'$. From this quadratic equation, the value of is found to be 3 miles;

$$\therefore \text{ Traction} = 125 \text{ lbs,}$$

and the load will be

$$125 \times 20 = 2500 \text{ lbs.}$$

*Ques.* 14. At what rate will a horse draw a ton, on a road whose coefficient of friction is $\frac{1}{32}$?

$$\text{Traction} = \frac{2240}{32} = 70 \text{ lbs.}$$

$$\therefore \quad 70 = 250 - (41\tfrac{2}{3})\, r;$$

$$\therefore \quad r = \frac{250 - 70}{41\frac{2}{3}} = 4.32,$$

the rate is 4.32 miles an hour.

*Ques.* 15. When a horse exerts a traction of $41\frac{2}{3}$ lbs., his rate of motion is 5 miles an hour, what gross load will he draw on a level road whose coefficient of friction is $\frac{1}{30}$, and what work will he perform a minute?

$$\text{Load} = 30 \times 41\tfrac{2}{3} = 1250 \text{ lbs.}$$

Distance moved in feet per minute,

$$= \frac{5 \times 5280}{60} = 440 \text{ feet.}$$

$$\text{Work} = 41\tfrac{2}{3} \times 440 = \mathbf{18333\frac{1}{3}}'.$$

*Ques.* 16. What must be the horse power of an engine, to cut 6600 square feet of planking in a day of 10 hours long?

It requires according to experiment, about 29000 units of work to saw a square foot of green oak planking, therefore,

The work in cutting 6600 square feet,

$$= 29000 \times 6600 = 191400000.$$

$$\therefore \frac{191400000}{10 \times 60} = 319000',$$

and $$\frac{319000'}{33000'} = 9\tfrac{2}{3}\text{ h.}$$

*Ques.* 17. An engine of 24 effective horse power, cuts 144 square feet of American live-oak in 5 minutes, how-many units of work are consumed in cutting a square foot?

The work of the engine in five minutes,

$$= 33000' \times 5 \times 24 = 3960000,$$

the work destroyed in cutting 144 square feet.

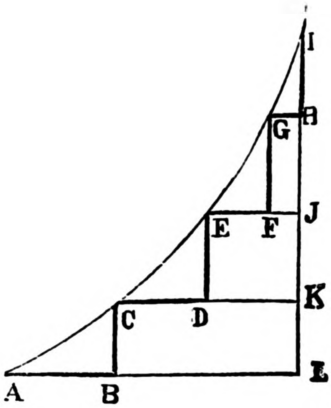
Therefore the units of work expended in cutting one square foot,

$$= \frac{3960000}{144} = 27500.$$

## CHAPTER III.

### *On the moving of bodies on inclined planes, and the raising of materials.*

If a surface be supposed without friction, the units of work performed by moving a body along it, is equal to the product of the weight of the body in pounds, by the vertical height in feet through which it is raised.




To move a body along the path A C E G I, may be imagined the same as to move it along and up an infinite number of steps, resembling A B, B C, C D, D E, &c. And since friction is neglected, there is no work required to move the body in a horizontal direction, all the work is expended in raising the body in the perpendicular direction; hence the units of work required to raise a body from L to I, equals the units of work required to move it along ths path A C E G I, the friction of the path being neglected.

The principle here explained holds true for an inclined plane.

## EXAMPLES.

*Question* 1. A TRAIN of 200 tons ascends an incline which has a rise of 5 feet in 1000, with a uniform speed of 30 miles an hour, what is the effective power of the engine, the friction being 5½ lbs. to the ton?

The pressure of a body on an inclined plane is nearly equal to its weight, when the inclination is small, hence the work due to friction, may be found bv the method explained in the last chapter.

Speed of train per minute = 2640 feet.

Weight of train in lbs. = 448000.

$$\frac{5}{1000} = \frac{1}{200}, \text{ rise}$$

of the rail in one foot.

$$\frac{1}{200} \times 2640 = 13.2 \text{ feet},$$

the rise of the rail in 2640 feet.

Consequently the whole weight of the train is raised 13.2 feet every minute in opposition to gravity.

Work done to gravity per minute,

$$= 448000 \times 13.2 = \mathbf{5913600'}.$$

Work done to friction per minute,

$$= 200 \times 5\tfrac{1}{2} \times 2640 = \mathbf{2904000}.$$

Total work of the engine per minute,

$$= \mathbf{5913600'} \times \mathbf{294000} = \mathbf{8817600'}.$$

$$\frac{\mathbf{8817600'}}{\mathbf{33000}} = \mathbf{267}^{\text{h}}.2.$$

A

*Ques.* 2. A train of 330 tons, ascends an incline that has a rise of $\frac{1}{5}$ in 100, what is the maximum speed with an engine of 120 horse power; the friction of the rail 8 lbs. per ton?

Let $x$ be the speed of the train in feet per hour.

The rise in each foot of rail,

$= \frac{1}{5}$ divided by $100 = \frac{1}{500}$;

the rise of the rail in $\times$ feet,

$$= \frac{x}{500};$$

The work due to gravity in an hour,

$$= 330 \times 2240 \times \frac{x}{500} = \mathbf{1478.4}\, x;$$

Work due to friction in an hour,

$$330 \times 8 \times x = \mathbf{2640}\, x.$$

It is evident that the work due to gravity in an hour, added to the work due to friction in an hour, must be equal to the work done by the engine in an hour,

$$\therefore \quad \mathbf{33000'} \times 120 \times 60 = \mathbf{2640}\, x \times \mathbf{1478.4}\, x,$$

$$\therefore \quad x = \frac{\mathbf{237600000}}{\mathbf{4118.4}} = 57692 \text{ feet.}$$

*Ques.* 3. A engine of 50 horse power ascends a gradient having a rise of $\frac{3}{4}$ in 100; with a steady speed of 20 miles an hour, what is the weight of the train in tons, the friction per ton being 8 lbs.?

Let $x$ be the weight of the train in tons; the work required to overcome friction,

$$= 1760 \times 8 \times x = \mathbf{14080}\, x' ;$$

since a speed of 20 miles an hour = 1760 feet a minute

Rise of rail in a foot,

$$= \tfrac{3}{4} \text{ divided by } 100 = \tfrac{3}{400} ;$$

Rise of the rail in 1760 feet,

$$= \frac{5280}{400} = 13.2 \text{ feet} ;$$

Work due to gravity,

$$= 2240 \times x \times 13.2 = \mathbf{29568}\, x'$$

Work of the engine per minute,

$$= \mathbf{33000}' \times 50 = \mathbf{165000}',$$

Consequently,

$$\mathbf{14080}\, x' + \mathbf{29568}\, x' = \mathbf{1650000}'$$

$$\therefore \quad x = \frac{\mathbf{1650000}}{\mathbf{43648}} = 37.8 \text{ tons.}$$

*Ques.* 4. A train of 100 tons *descends* a gradient having a rise of $\frac{1}{4}$ of a foot in 100 feet, at a uniform speed of 60 miles an hour, what is the horse power of the engine, friction reckoned at 8 lbs. a ton?

60 miles an hour = 5280 feet a minute ;

Work due to friction,

$$= 100 \times 5280 \times 8 = \mathbf{4224000}'.$$

Rise in one foot,

$$= \tfrac{1}{4} \text{ divided by } 100 = \tfrac{1}{400}.$$

Rise in 5280 feet, or in one minute,

$$= \frac{5280}{400} = 13.2 \text{ feet.}$$

Work due to gravity,

$$= 224000 \times 13.2 = \mathbf{2956800}'.$$

In this case gravity acts with the engine ;

$$\therefore \quad \mathbf{4224000}' \text{ minus } \mathbf{2956800}' = \mathbf{1267200}'$$

$$\frac{\mathbf{1267200}'}{\mathbf{33000}'} = \mathbf{38}^{\text{h}}.\mathbf{4}.$$

*Ques.* 5. If a horse exerts a traction of 144 lbs., what weight can he pull on a plank road, up a hill that has a rise of 3 feet in 190 feet, supposing the coefficient of friction to be $\frac{1}{20}$?

Work of the horse in moving over 190 feet,

$$= 190 \times 144 = \mathbf{27360}.$$

The work of friction in moving $x$ lbs. over 190 feet,

$$\frac{x}{20} \times 190 = \mathbf{9.5}\, x,$$

supposing $x$ to be the required load;

The work due to gravity, when the load is moved over 190 feet $= \mathbf{3}\, x$;

$$\therefore \quad \mathbf{9.5}\, x + \mathbf{3}\, x = \mathbf{27360};$$

$$\therefore \quad x = \frac{\mathbf{27360}}{\mathbf{12.5}} = 2188.8 \text{ lbs.}$$

*Ques.* 6. What would be the backward pressure of a horse in going down hill, that has a rise of 15 feet in 369 with a load of 2000 lbs. supposing the coefficient of friction to be $\frac{1}{30}$?

Work due to gravity in moving 2000 lbs. 369 feet,

$$= 2000 \times 15 = \mathbf{30000};$$

Work due to friction in moving the load 369 feet,

$$= \frac{2000}{30} \times 3.69 = \mathbf{24600};$$

From **30000**
Take **24600**
**5400**

$$\therefore \quad \frac{\mathbf{5400}}{\mathbf{369}} = 14.6 \text{ lbs.}$$

*Ques.* 7. How many horses would it take to draw a load of 6 tons up a hill having a rise of $2\frac{1}{2}$ in 100, supposing the resistance of friction to be $\frac{1}{12}$ of the whole load, and the traction of each horse, 160 lbs.

Let $x$ be the number of horses,

Work due to friction in moving 6 tons over 100 feet,

$$= \frac{6 \times 2240}{12} \times 100 = \mathbf{112000}.$$

Work of gravity in moving 6 tons over 100 feet,

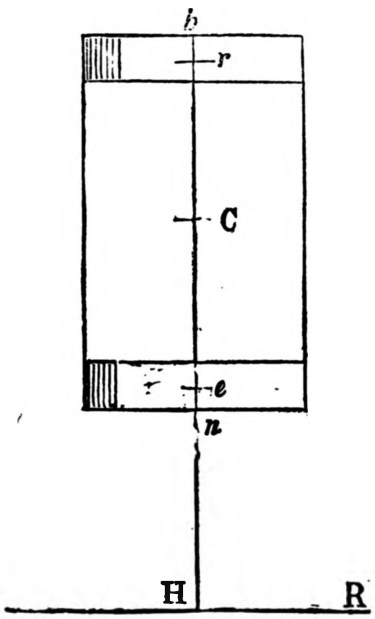
$$= 6 \times 2240 \times 2\tfrac{1}{2} = \mathbf{33600}.$$

The work of $x$ horses in passing over 100 feet of this road,

$$= 160 \times x \times 100 = \mathbf{16000}\,x;$$

$$\therefore \mathbf{16000}\,x = \mathbf{112000} + \mathbf{33600},$$

$$\therefore x = \frac{\mathbf{145600}}{\mathbf{16000}} = 9.1 \text{ horses.}$$




The work in raising materials, having a given form, will be its weight multiplied by the height to which the centre of gravity is raised.

Suppose the body $n\,b$ to be raised from the horizontal line H, R, and the point C to be the middle or centre of gravity of the body; let $e$ and $r$ be points equi-distant from C. Now if equal weights, say for example 3 lbs. be placed at $e$ and $r$, the centre of gravity of these weights will be at C, then

Work in raising 3 lbs. to $r = 3 \times \mathrm{H}\,r$.

" " " to $e = 3 \times \mathrm{H}\,e$.

Sum $= 3 \times (\mathrm{H}\,r + \mathrm{H}\,e)$.

But $\mathrm{H}\,r + \mathrm{H}\,e = 2\,\mathrm{H\,C}$;

$\therefore$ sum $= 6 \times \mathrm{H\,C}$.

That is, 6 lbs. raised from H to C is the same amount of work as 3 lbs. raised to $r$, and 3 lbs. raised to $e$. The same may be proved of any two equal weights, one of them raised as far above the centre of gravity C, as the other is below it.

*Ques.* 8. Required the units of work in raising the material of a wall 22 feet long, 13 feet high, and $2\frac{1}{2}$ feet thick; supposing the weight of a cubic foot of the material to be 140 lbs.?

Contents of the wall in cubic feet,

$$= 22 \times 13 \times 2\tfrac{1}{2} = 715.$$

Weight of the wall in pounds,

$$= 715 \times 140 = 10010.$$

The height of the centre of gravity of the whole wall,

$$= \frac{13}{2} = 6\tfrac{1}{2} \text{ feet};$$

$$\text{Work} = 10010 \times 6\tfrac{1}{2} = \mathbf{65065}.$$

*Ques.* 9. The shaft of a pit is to be sunk 120 feet deep, and 6 feet in diameter; in how many days would a man working with a wheel and axle, raise the material, supposing a cubic foot to weigh 100 lbs.?

Number of cubic feet in the shaft,

$$= 6^2 \times .7854 \times 120 = 3392.928;$$

Weight of the material,

$$= 3392.928 \times 100 = 339292.8 \text{ lbs.}$$

The units of work to raise this material $\frac{120}{2}$ feet,

$$= 339292.8 \times \frac{120}{2} \quad \mathbf{20357568};$$

for it is clear that the centre of gravity of the shaft is 60 feet from the top.

A man will perform $\mathbf{2600}'$ working 8 hours a day, turning a handle.

Units of work in 8 hours,

$$= \mathbf{2600}' \times 60 \times 8 = \mathbf{1248000}.$$

The number of days required,

$$= \frac{\mathbf{20357568}}{\mathbf{1248000}} = 16.33.$$

*Ques.* 10. Required the work in raising 3 cwt. of coals from a pit whose depth is 120 feet, the circumference of the rope being two inches, allowing the weight of one foot of the rope of 1 inch in circumference to be .046 lbs.?

Weight of one foot of rope,

$$= 2^2 \times .046 = .184 \text{ lbs.}$$

Weight of the whole rope,

$$= .184 \times 120 = 22.08 \text{ lbs.}$$

Weight in raising the rope,

$$= 22.08 \times \frac{120}{2} = \mathbf{1324.8}.$$

Work in raising the coals,

$$= 112 \times 3 \times 120 = \mathbf{40320};$$

Total work,

$$= \mathbf{40320} \times \mathbf{1324.8} = \mathbf{41644.8}.$$

*Ques.* 11. A cistern 22 feet long, 10 feet broad, and 8 feet deep; required the work in filling it, when the height of the bottom of the cistern from the water in the well is 36 feet?

Water in the cistern,

$$= 22 \times 10 \times 8 \times 62.5 = 110000 \text{ lbs.}$$

The height to which the centre of gravity of the water has to be raised,

$$= \frac{8}{2} + 36 = 40 \text{ feet.}$$

Work $= 110000 \times 40 = \mathbf{4400000}$.

*Ques.* 12. The side A B of a cube of granite is 6 feet, and the weight of a cubic foot is 170 lbs.; it is required to find the work necessary to overturn it on the edge of A?

The distance of the centre of gravity $g$, from the edge A,

$$= \sqrt{\frac{6^2}{2}} = 4.24 \text{ feet.}$$

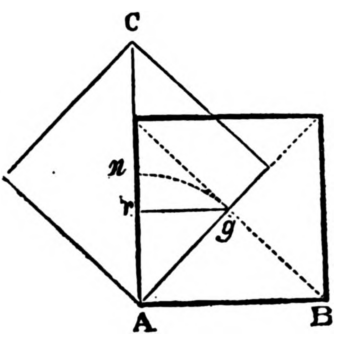
When the centre is about to fall, the centre of gravity is raised from $r$ to $n$, consequently the work in overturning

the body is the same as raising its whole weight a perpendicular height = $r\ n$,

$$\therefore\ r\ n = 4.24 - 3 = 1.24.$$

$$\text{Work} = 170 \times 6^2 \times 1.24 = \mathbf{45532.8}.$$

In a body of any form about to fall, the centre of gravity $g$, will be at $n$ in the vertical line A C, the work in bringing the body to this position, is due to the vertical distance $r\ n$, through which the centre of gravity has been raised. The work necessary to overturn any body is the true measure of stability.



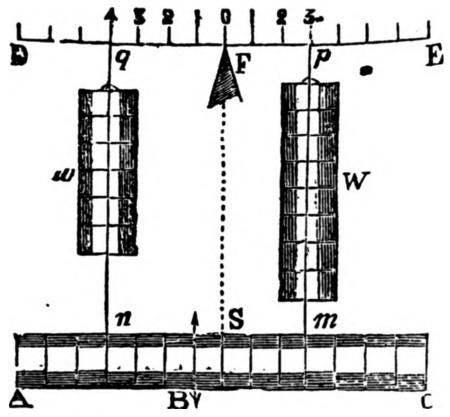

## CHAPTER IV.

### ON THE LEVER, WHEEL AND PULLEY, INCLINED PLANE, WEDGE AND SCREW.

In this chapter I intend to treat of the transmission of work by simple machines, it may be observed, that the object of machinery, whether simple or compound, is to regulate the distribution of work, or to change the direction—and not to increase work.

If the parts of a machine were not subject to friction or any other resistances, the *work* given out would be exactly equal to the work applied. Dead matter, by its gravity, produces pressure, and by the intervention of mechanism that pressure may be increased or decreased; but *work* is peculiarly the production of active or living agents.

## THE LEVER.




Suppose two uniform bars A B, B C, to be suspended from their centres *w* and *m*, by means of chords attached to the points 4 and 3 of the lever D E turning on the fulcrum F, which must evidently be in the middle of D E = A C, and over the middle of A C, in order to secure equilibrium; that is, in order that the two parts may rest horizontally as if they were in one piece, and suspended by the middle point S. It is also evident that the equilibrium will not be destroyed if the bars be hung by their ends at the points 4 and 3.

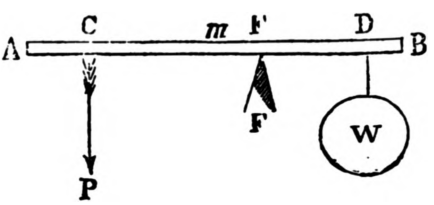
Let the weight of the bar A B = 6 lbs., and the weight of B C = 8 lbs., then A B will contain 6 units of length, and B C 8 units of length. It is evident from the figure, that F *q*, the distance at which A B acts from the fulcrum contains 4 units, and F *p*, the distance at which B C acts from the fulcrum will contain 3 units; then since it appears that a weight of 6 lbs. suspended at *q*, balances a weight of 8 lbs., suspended at *p*; therefore the following relation exists when equilibrium takes place;

$$6 \text{ lbs.} \times 4 = 8 \text{ lbs.} \times 3, \text{ generally}$$

$$w \times q\,\mathrm{F} = \mathrm{W} \times \mathrm{F}\,p.$$

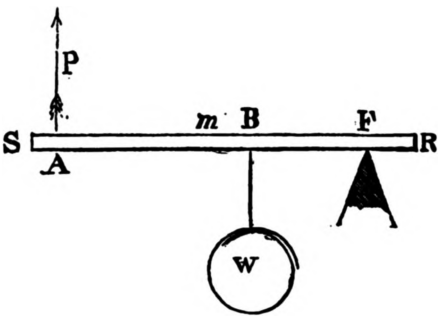
Levers are divided into three kinds:—In the *first kind of lever*, the power and weight are on opposite sides of the fulcrum.

**First order of Lever.**




A B is a lever of the first order, F the fulcrum or prop, P the power acting at A C, and W the weight attached to the point D. If C F be twice D F, 5 lbs. at C will balance 10 lbs. at D; generally as many times as C F is longer than F D, so many times will W be greator than P.

**Second order of Lever.**

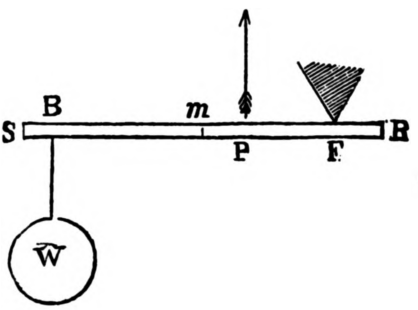



Lever of the second kind.—The weight is between the fulcrum and the power. W is the weight, E the fulcrum, P the power.

When the lever is supposed to be without weight, then if the length of A F $= a$, and B F $= p$, the power P balances the weight W when,

$$W \times p = P \times a.$$

**Third order of Lever.**




Third kind or order of Lever.—In this case the power is between the fulcrum and the weight. P, represents the power, F the fulcrum, and W the weight. Generally if B F $= p$, and P F $= q$, the power P balances the weight W, when

$$W \times p = P \times q.$$

EXAMPLES.

*Question* 1. In a lever of the first kind, let W $=$ 30 lbs. The arm C F $= 11$ feet, the arm F D $= 4$; required P?

Put $x =$ P in lbs.

Then by the equality of moments,

$$11 \times x = 4 \times 30,$$

$$\therefore\ x = \frac{120}{11} = 10\tfrac{10}{11} \text{ lbs.}$$

*Ques.* 2. A man exerts a pressure of 50 lbs. on a crowbar at a distance of 4 feet from the fulcrum, what weight will he balance at the distance of 3 inches from the fulcrum?

$$W \times 3 = 50 \times 48,$$

$$\therefore\ W = 800 \text{ lbs.}$$

*Ques.* 3. In a lever of the second kind, W = 11 lbs., B F = 16 inches, and A F = 100 inches; required P?

$$P \times 100 = 11 \times 16,$$

$$\therefore \quad P = 1.76 \text{ lbs.}$$

*Ques.* 4. In a lever of the third kind, W = 40; B F = 60 inches, and P F = 8; required P?

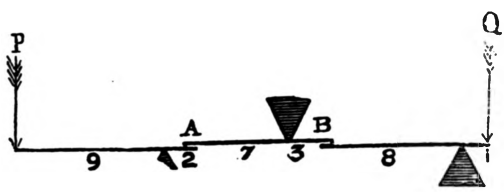
$$P \times 8 = 40 \times 60,$$

$$\therefore \quad P = 300 \text{ lbs.}$$

*Ques.* 5. In a lever of the first kind, 5 and 8 lbs. are placed on one side of the fulcrum, at a distance of 6 and 4 inches respectively from the fulcrum; required the power P, acting at the distance of 9 inches from the fulcrum, to maintain equilibrium?

$$P \times 9 = 5 \times 6 + 8 \times 4,$$

$$\therefore \quad P = \frac{62}{9} = 6\frac{8}{9} \text{ lbs.}$$




*Ques.* 6. In a combination of three levers of the first order, represented on the accompanying diagram, the long arms are 9, 7, 8, inches respectively, the short arms 2, 3, 1 inches; If a pressure of 10 lbs. be applied at P, what is the pressure at Q to balance it?

$$9 \times 10 = 2 \times \text{the pressure at A,}$$

The pressure at A = 45 lbs.,

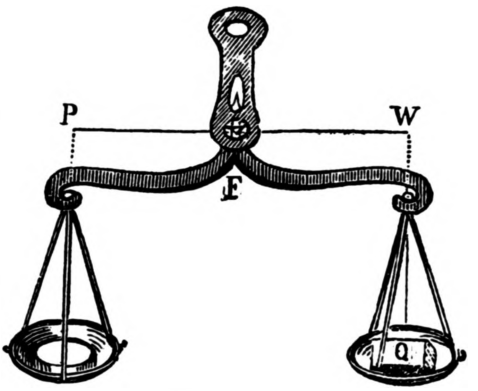
$$7 \times 45 = 3 \times \text{the pressure at B,}$$

Pressure at B = 105 lbs.

$$105 \times 8 = 1 \times \text{the pressure at Q.}$$

Pressure at Q = 840 lbs.

*Ques.* 7. Let P F and F W be the arms of a false balance, a certain weight Q weighs 16 lbs. when put in the scale attached to the long arm, and only 9 lbs. when weighed in the opposite scale; what is the true weight of Q?




By the equality of moments, the two following equations are obtained.

$$Q \times W F = 9 \times P F;$$
$$Q \times P F = 16 \times W F.$$

Multiplying these equations together, and then striking out the common factors,

$$Q^2 = 9 \times 16;$$
$$\therefore \quad Q = 12.$$

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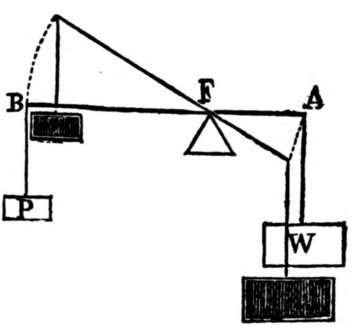
### THE PRINCIPLE OF WORK APPLIED TO THE LEVER.

Let B F = 10 feet, F A = 2 feet, and P = 3 lbs.; required W?

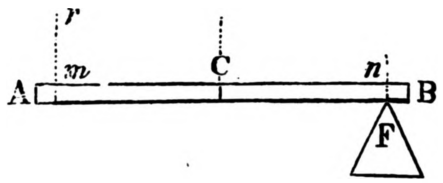



It is evident, if P be raised 5 feet, W will be depressed one, because F B is five times the length of F A; consequently the work of P = 3 × 5, and the work of W = W × 1;

$$\therefore \quad W \times 1 = 3 \times 5;$$
$$\therefore \quad W = 15 \text{ lbs.}$$

Hence the equality of moments is readily established by the principle of work, and conversely. When the motion is extremely small, the principle of work is termed the principle of virtual velocities.

## OF THE LEVER WHEN ITS WEIGHT IS TAKEN INTO ACCOUNT.




The tendency of a uniform beam or lever A B, to turn about the fulcrum F, is just the same as if the whole of its weight were collected in its middle point, or centre of gravity C. For if the preponderating side A F, be hung from a cord $m\,r$, placed so that A $m$ = $n$ B, then this cord will sustain one half the weight of the beam. Now if the whole weight of the beam be collected at C, the centre, it would produce the same strain upon the cord, hence the beam acts as if its whole weight were collected in its middle point.

*Ques.* 8. The weight of the lever of the first kind = 10 lbs., the length A B = 56 inches; A F = 40; C F = 36; D F = 5 inches, and W = 150 lbs; required P, in order to maintain the lever in equilibrium. In this case the weight of the lever acting at its centre of gravity $m$, co-operates with the power,

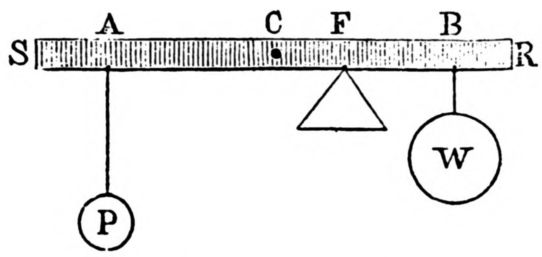
$$m\,F = A\,F - A\,m,$$

$$= 40 - \tfrac{1}{2} \text{ of } 56 = 12$$

Then by the equality of moments,

$$P \times 36 + 10 \times 12 = 5 \times 150.$$

$$P = 17.25 \text{ lbs.}$$




*Ques.* 9. The weight of the lever S R, of the first order = 31 lbs.; S R = 85 inches; S F = 55; A F = 43; B F = 19; and W = 34 lbs. Required P?

mmm mM

S F = 55.

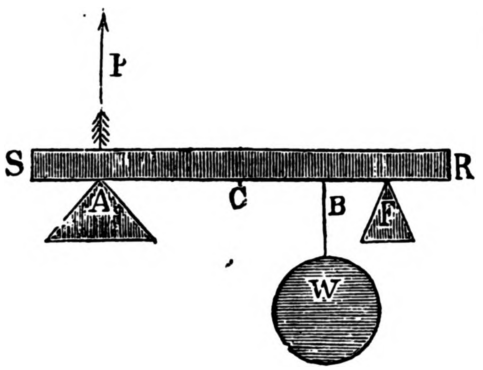
$$S\,C = \frac{85}{2} = 41\tfrac{1}{2}$$

C F = $3\frac{1}{2}$;

$$\therefore\ P \times 43 + 31 \times 3\tfrac{1}{2} = W \times 19,$$

$$\therefore\ P = \frac{34 \times 19 - 31 \times 3\frac{1}{2}}{43} = 12\tfrac{1}{2} \text{ lbs.}$$

*Ques.* 10. The weight of a lever of the second order = 8 lbs. S R = 80 inches; S F = 76; A F = 70 inches; B F = 2 inches, and P = 100 lbs.; Required W?




In this case, the weight of the lever acts with W;

$$\therefore\ 76 - \frac{80}{2} = 36$$

$$W \times 2 + 8 \times 36 = 100 \times 70,$$

$$\therefore\ W = 3356 \text{ lbs.}$$

*Ques.* 11. A beam R S, whose weight is 4 cwt., is supported by props at A and F; a weight W, of 20 cwt. is placed at B; it is required to determine the pressures on the props, when R S = 50 feet; F R = 2 feet; B F = 12 feet; A F = 30 feet?

Let C be the centre of the beam, then C R = $\frac{1}{2}$ of 50 = 25; C F = 25 — 2 = 23; suppose the beam to turn on F as a fulcrum, the pressure on A,

$$= P \times 30 = 20 \times 12 + 4 \times 23,$$

$$\therefore\ P = \frac{33^2}{30} = 11\ \frac{2}{30} \text{ cwt.}$$

Because the two props support the whole weight 24 cwt., 24—$11\frac{2}{30}$ = $12\frac{14}{15}$ cwt. the pressure on F.

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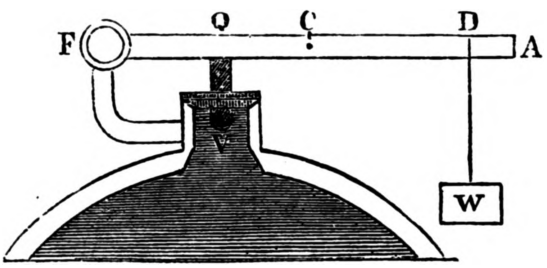
## HOW TO GRADUATE THE LEVER OF A SAFETY VALVE OF A STEAM ENGINE.

A F is a graduated lever of the second kind, turning upon F as a centre; V the valve of opening or closing, as the

case may be, the communication of steam in the boiler with the atmosphere; the lever A F rests upon the pin Q V of the valve V, and a sliding weight W is suspended from the lever, enabling the engineer to place any amount of pressure on the valve; this pressure measures the elasticity of the steam when it begins to escape. When steam of very high temperature is employed, the admission of atmospheric air is dangerous, and under such circumstances this valve may be properly termed the unsafety valve. In order to graduate the lever, a weight W must be found, so that when it is placed at A it may balance the greatest pressure of the steam.

The next thing to be found, is the position of the weight W, to give any proposed pressure to the valve.




EXAMPLES.

*Ques.* 12. The length A F of a lever =14 inches; the distance F Q = 2 inches; the weight of the valve and pin = 5 lbs.; the weight of the lever F A = 8 lbs., and the area of circular value in the narrowest place = 6 square inches; the it is required to find the load W, so that when it is placed at the extremity of the lever, the steam may have a pressure of 30 lbs. on the square inch, or 45 lbs. including the pressure of the atmosphere?

Pressure of steam on valve,

$$= 6 \times 30 = 180 \text{ lbs.}$$

Hence the effective pressure on the lever,

$$= 180 - 5 = 175 \text{ lbs.},$$

since the weight of the lever will act at the middle point C,

$$W \times 14 + 8 \times 7 = 175 \times 2.$$

$$\therefore \quad W = 21 \text{ lbs.}$$

*Ques.* 13. At what distance from the fulcrum must the load be, in the last example, so that the steam may have a pressure of 16 lbs. over the pressure of the atmosphere?

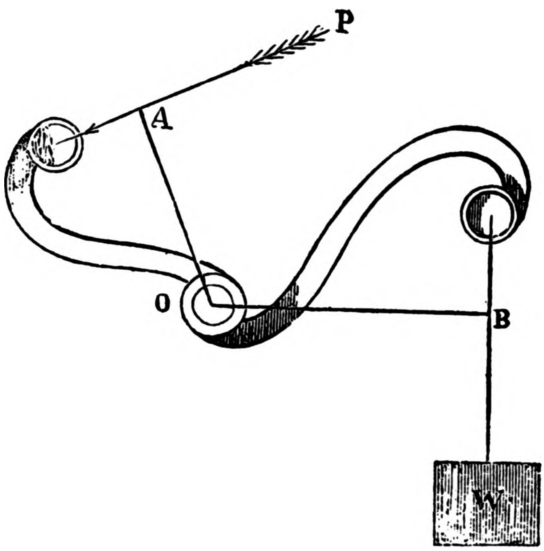
$$16 \times 6 - 5 = 91.$$

Suppose D to be the required position of the load, then

$$F D \times 21 + 8 \times 7 = 91 \times 12;$$

$$\therefore \quad F D = 6 \text{ inches.}$$

*Ques.* 14. In a bent lever the perpendicular O A, on the direction of the force P is 4 inches; and the perpendicular O B upon the direction of W is 7 inches; Required P when W = 200 lbs.?

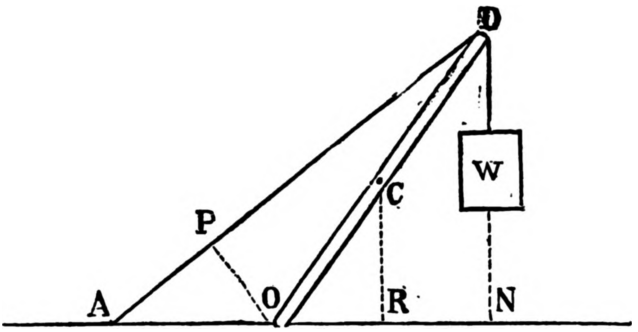



$$P \times 4 = 200 \times 7,$$

$$\therefore \quad P = 350.$$

*Ques.* 15. A rope A D supports a uniform pole O D, resting on the ground at O, and supporting the weight W, suspended from D; required the tension of the rope, when A D = 175 feet, O A = 40 feet, O D = 145 feet, W = 50 cwt., and weight of O D = 10 cwt?

In this example O D may be regarded as a lever turning on O as a centre. Draw O P perpendicular to A D, and C R a vertical line passing through the centre of gravity C of the pole, then the moment of the force stretching the




rope, will be equal to the sum of the moments of W and the weight of the pole, that is

Tension of cord $\times$ O P = W $\times$ O N + Wt. of pole $\times$ O R.

To find the perpendiculars,

O P, D N, C R,

in order to do this in the most simple manner, it will be observed, that since the three sides of the triangle A O D are given, the area may be found by the common rule; the area = 2100 square feet; but the area is also expressed by

$$\frac{A D \times O P}{2} = 2100,$$

$$\therefore \quad O P = 24 \text{ feet.}$$

But the area is also =

$$\frac{A O \times D N}{2} 2100,$$

$$\therefore \quad D N = 105 \text{ feet.}$$

$$O N = \sqrt{145^2 - 105^2} = 100,$$

$$\therefore \quad O R = 50.$$

$\therefore$ Tension of cord $\times$ 24 = 50 $\times$ 10 + 10 $\times$ 50.

Consequently the tension of the cord

$= 229\frac{1}{6}$ cwt.

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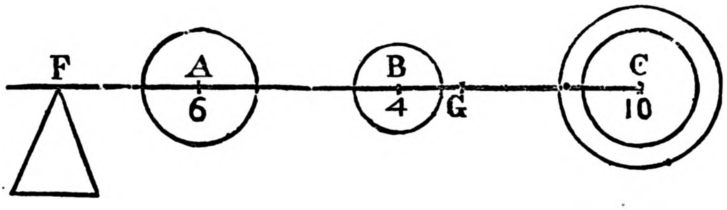
## THE CENTRE OF GRAVITY.

Every heavy body is composed of an infinite number of particles, each of which is acted on by the force of gravity in a direction perpendicular to the horizon. The sum of

all the parallel forces is evidently the weight of the body.

Now there must be a point, where a single force, equal to the weight of the body, being applied, will produce the same effect, as the force of gravity acting upon the various particles composing the body ;—this point is called the centre of gravity of the body. From this deffinition it immediately follows ; that if the centre of gravity be supported, the body will stand, and *vice versa*. That the centre of gravity of all symetrical, or regular bodies is in their centre of magnitude. That if any body, acted on by the force of gravity tend to turn a lever, we may regard the weight of the whole body to be collected in its centre of gravity.




*Ques.* 16. Let A, B and C, be three bodies in the same right line, it is required to determine the position of their common centre of gravity G with respect to any assumed point F, when A = 6 lbs., B = 4 lbs., C = 10 lbs. ; A F = 10 feet, B F = 25 feet, and C F = 42 feet?

Let it be supposed that an inflexible rod, without weight, passes through the bodies, and that the system turns upon F as a fulcrum ; then as the whole mass may be regarded as acting in the point G, by the equality of moments.

$$F\,G\,(6 + 4 + 10) = 6 \times 10 + 4 \times 25 + 10 \times 42,$$

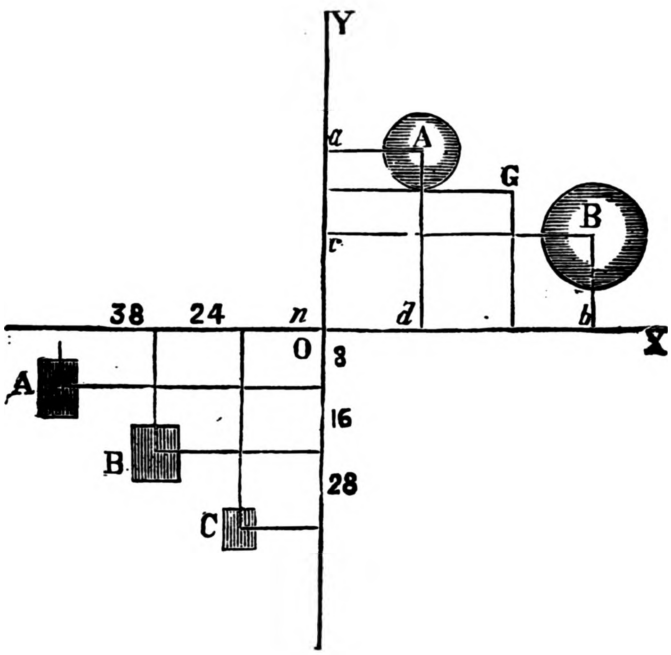
$$\therefore\ F\,G \times 20 = 580,$$

$$F\,G = 29 \text{ feet.}$$

Let A, B, &c., being any number of bodies lying in the same horizontal plane ; it is required to determine the position of the centre of gravity G, referred to two axes or lines, O X, O Y, perpendicular to each other, these lines O X and O Y are termed co-ordinate axes.

Conceive the bodies to be connected with the axis O X, by the perpendicular rods A *d*, B *b*, &c., then the sum of the moments of the bodies, tending to turn round the axis of

O X, will be the same as the moment of the whole mass, collected in the centre of gravity G. From this equality the distance of G from O X is obtained. In precisely the same way, we find the distance of G from O Y ; and hence




the point G becomes known. After the same manner the centre of gravity may be determined, when the bodies are in space, by referring them to the co-ordinate planes.

*Ques.* 17. The weights of three bodies A, B, C, are 10, 17, and 9 lbs. respectively ; and their distances from O X are 8, 16 and 28 inches, and from O Y, 32, 24 and 11 inches respectively ; required the position of the common centre of gravity ?

Let $x$ be the distance of the centre of gravity from the O X ; and $y$ the distance from O Y, then

$$(10 + 17 + 9)\, x = 10 \times 8 + 17 \times 16 + 9 \times 28,$$

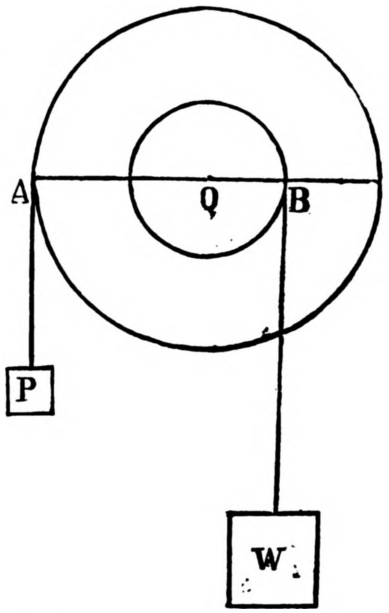
$$\therefore \quad x = 14.$$

$$(10 + 17 + 9)\, y = 10 \times 38 + 17 \times 24 + 9 \times 11,$$

$$y = 24\ \frac{23}{36}.$$

## WHEEL AND AXLE.

This useful machine is only another form of the lever, where the power is made to act with intermission; in its most simple form, it consists of a large wheel A, and a cylinder, or axle, B, both of which turn on the same axis O. If the wheel A, be turned round by a power P, applied to it, the axle B, will coil up the rope by which the weight W hangs. The lever by which P acts, is evidently A O, and that by which W acts is O B; hence when these weights, or pressures, balance each other.




$$P \times A\,O = W \times O\,B.$$

If the wheel be displaced by a handle, then the machine is called a windlass. Sometimes the handle is made to turn a series of wheels acting on each other by means of teeth; when a machine has this form, it is called a crane.

I will now consider the equilibrium, on the principle of work.

When the diameter of a circle = 1, the circumference = 3.1416.

When the wheel makes 1 revolution, the axle also makes one. In one turn P descends a space,

$$= 2 \times O\,A \times 3.1416,$$

and W ascends a space

$$= 2 \times O\,B \times 3.1416.$$

The work done by P in one revolution

$$= 2 \times A\,O \times 3.1416 \times P;$$

and the work of W in one revolution,

$$= 2 \times O\,B \times 3.1416 \times W.$$

It is clear that the work done in one revolution is equal to the work applied, friction being neglected,

$$\therefore\ 2 \times AO \times 3.1416 \times P = 2 \times OB \times 3.1416 \times W.$$

$$\therefore\ P \times OA = W \times OB,$$

which is the relation already established, conversely assuming the equation of equilibrium, the principal of work is readily established.

*Ques.* 18. The handle of a windlass is 18 inches, the radius of the axle = 3 inches, and the power applied = 60 lbs.; what weight could be raised were there no friction?

Work of P in one revolution,

$$= 60 \times \frac{2 \times 18}{12} \times 3.1416.$$

Work of W in one revolution,

$$= W \times \frac{2 \times 3}{12} \times 3.1416.$$

$$\therefore W \times \frac{2 \times 3}{12} \times 3.1416 = 60 \times \frac{2 \times 18}{12} \times 3.1416$$

$$\therefore W = 360 \text{ lbs.}$$

*Ques.* 19. Required W in the last example, when the thickness of the cord is 1 inch, supposing that $\frac{1}{8}$ of the work applied to be lost in friction, and the rigidity of the cord?

The cord increases the radius of the wheel $\frac{1}{2}$ of an inch, hence

Work of W in one revolution,

$$= W \times \frac{7}{12} \times 3.1416;$$

Effective work of P in one revolution,

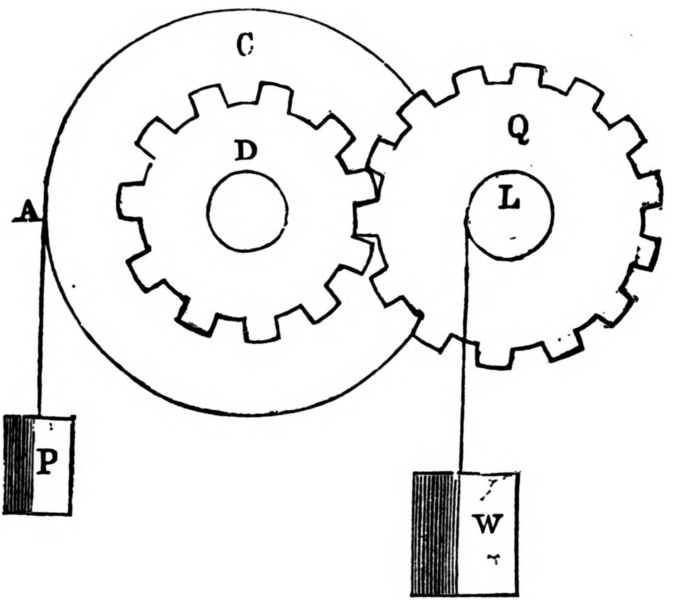
$$= \frac{7}{8} \times 60 \times 3 \times 3.1416,$$

$$\therefore\ W \times \frac{7}{12} \times 3.1416 = \frac{7}{8} \times 60 \times 3 \times 3.1416.$$

$$\therefore\ W = 270 \text{ lbs.}$$

## OF COGGED OR TOOTHED WHEELS.

Let D be a cogged wheel turning upon the same axis as the wheel C ; Q another cogged wheel, acted upon by the former and turning upon the same axis as the axle L. From the wheel C is suspended the weight P, and from the axle L the weight W ; then while P descends, the wheel C and the cogged wheel D, will be turned from right to left,




but as every tooth in the cog-wheel D is being turned round, a corresponding tooth in the cog-wheel Q will be turned in the contrary direction, and thus the cord L W will be coiled up on the axle L, and the weight W will be raised.

*Ques.* 20. Let P = 120 lbs., the diameter of the wheel C = 3 feet, the number of teeth in D = 11, the number in Q = 14, the diameter of the axle L = $\frac{1}{2}$ a foot ; required W, in order that equilibrium may take place ?

For every turn of the wheel C, 11 teeth of the wheel Q will be turned round. therefore as many times as the number of teeth in D can be taken out of the number in Q, so many turns will the cog-wheel D make while the wheel Q makes one.

Let the axle L and wheel Q make one revolution, then the revolutions made by the wheel C,

$= 14$ divided by $= \frac{14}{11}$.

The space moved over by W,

$$= \tfrac{1}{2} \times 3.1416\ ;$$

The space moved over by P,

$$\frac{14}{11} \times 3 \times 3.1416\ ;$$

Work due to $W = \frac{1}{2} \times 3.1416 \times W.$

Work due to $P = 3 \times 3.1416 \times \frac{14}{11} \times 120.$

∴ Since, the work due to P and W are equal during one revolution of L, neglecting friction,

$$\frac{1}{2} \times 3.1416 \times W = \frac{14}{11} \times 3 \times 3.1416 \times 120,$$

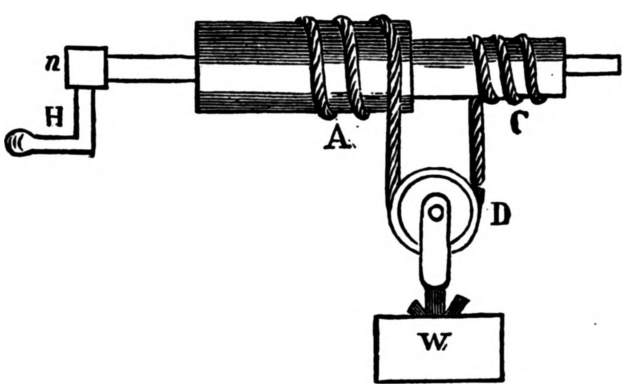
$$\therefore \quad \tfrac{1}{2}\, W = \tfrac{14}{11} \times 3 \times 120.$$

$$W = 916\ \tfrac{4}{11}\ \text{lbs.}$$

---

## COMPOUND WHEEL AND AXLE.

In the common wheel and axle, there is a practical limit to the power of the machine; for we can only increase the power by increasing the size of the wheel, or by decreasing the radius of the axle. But in the compound wheel and axle, a given power may be made to raise any weight whatever.




This machine consists of two axles, A and C, cut upon

the same block, round which a cord coils in opposite directions; this cord passes round the moveable pulley D, which carries the weight W. Now if the handle be turned, one of the cords is coiled upon the large axle A, while the other cord is uncoiled from the small axle C, so that the rate at which W ascends, depends upon the difference of the circumferences of the two axles; and, consequently, the power of the machine will also depend upon this difference. But this may be decreased to any extent, without altering the length of the handle. This truly ingenious contrivance is due to the Chinese.

*Ques.* 21. Let the diameter of the axle A, = .8 feet, the diameter of axle C, = .6 feet; the length of the handle H *h*, = $3\frac{1}{2}$ feet; and W, = 6090 lbs. Required P?

When the handle makes one turn, the cord A will be drawn up a space equal to the circumference of the axle A, while the cord C, will be let down a space equal to the circumference of the axle C; therefore the whole cord will be shortened a space equal to the difference of the circumferences, and because the cord is doubled, the weight W, will be raised a space only equal to the half of this difference.

$$\frac{.8 \times 3.1416 - .6 \times 3.1416}{2} \times 6090;$$

Work done upon P, in one revolution,

$$= 7 \times 3.1416 \times P;$$

$$\therefore \quad 7 \times 3.1416 \times P = \frac{.8 \times 3.1416 - .6 \times 3.1416}{2} \times 6090$$

Dividing each side by 3.1416,

$$\therefore \quad 7P = \frac{.8 - .6}{2} \times 6090,$$

$$P = 84 \text{ lbs.}$$

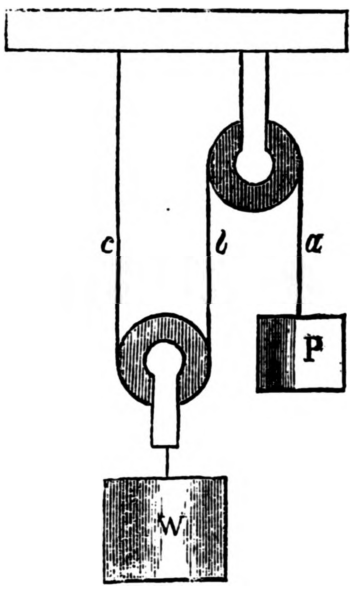
---

## THE PULLEY.

A pulley is a grooved wheel, turning on an axis, and

placed in a block or case. A cord passes over the groove of the wheel, in order to transmit the force applied, in any proposed direction. There is no advantage gained by a single fixed pulley; but when there are moveable pulleys, the weight raised will always be greater than the power applied; and then the advantage depends upon the number of cords by which the weight is suspended.

*Ques.* 25. In the annexed system of pulleys, if W = 500 lbs., required P, when equilibrium takes place?




As W is suspended by two cords, *c* and *b*, each cord will support 250 lbs.; but as the cord is supposed to have a free motion over the wheels, the portions *a*, *b*, *c*, will have the same stretch or tension; hence P = 250 lbs.

*Application of the Principle of Work.*

When W ascends 1 foot, the cords *c* and *b* will each be shortened 1 foot, and and therefore P must descend 2 feet. Hence the work of P

$$= P \times 2;$$

and the work of $W = W \times 1$;

$$\therefore \quad P \times 2 = W \times 1.$$

$$P = \frac{W}{2}.$$

*Ques.* 23. Let there be two moveable pulleys, each weighing 4 lbs., the if P = 60 lbs. it is required to determine W, on the principle of work?

If P descend 4 feet, the first moveable pulley will ascend 2 feet, and the second 1 foot.

Consequently, the work done in raising W and the pulleys =

$$W \times 1 \times 4 \times 1 \times 4 \times 3 = W \times 12;$$

The work of P $= 60 \times 4$;

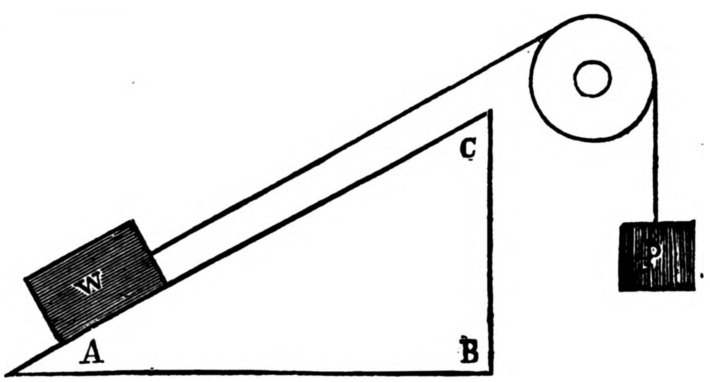
$$\therefore \quad W \times 12 = 60 \times 4.$$

$$W = 228 \text{ lbs.}$$

---

## OF THE INCLINED PLANE.

To find the pressure necessary to support a body on an inclined plane without friction. Let the weight W, be drawn up the inclined plane A C by means of the weight P, actiug by a cord parallel to the plane. then whilst W is moved from A to C, the counterpoise weight will have descended a vertical space equal to A C, according to the principle of work.




Work in raising W $= W \times CB$;

Work due to the descent of P

$$= P \times AC;$$

$$\therefore \quad P \times AC = W \times CB;$$

$$\therefore \quad P = \frac{CB}{AC} W.$$

*Ques.* 24. The length of an inclined plane is 30 feet, the perpendicular height 10 feet. and the weight of the body W = 20 cwt., what pressure P, will be required to sustain the body on the plane, friction being neglected?

Work in raising W in opposition to gravity,

$$= 20 \times 112 \times 10 = 22400;$$

The work of $P = P \times 30$;

$$\therefore \quad P \times 30 = 22400.$$

$$P = 746\tfrac{2}{3} \text{ lbs.}$$

*Ques.* 25. The length of an inclined plane is a mile or 5280 feet. the height 88 feet. the weight of the body 1760 lbs., and the friction $\frac{1}{264}$ part of the weight, what pressure will be required to move the body up the plane?

In this case the inclination of the plane is small, and hence the pressure upon it is very nearly equal to the weight of the body.

$\therefore$ Pressure to overcome friction $= \frac{1760}{264}$ lbs.

Work due to friction, =

$$\frac{1760}{264} \times 5280 = 35200;$$

Work due to gravity, =

$$1760 \times 88 = 154880;$$

Work of the pressure P applied to move the body,

$$= P \times 5280;$$

$$\therefore \quad P \times 5280 = 35200 + 154880.$$

$\therefore$ P = 36 lbs.

---

## OF THE WEDGE, OR MOVEABLE INCLINED PLANE.

Let A B C be a wedge, (*see the last figure,*) sliding on the horizontal plane A B, by the action of a pressure P, applied parallel to A B, and thereby elevating a weight W which is only free to move in a vertical direction. When

the wedge begins to act, the resistance W, rests upon the horizontal plane A B ; but when the wedge has moved over a space equal to its length, W will have been elevated a height equal to the thickness C B of the wedge ; then if A B = 1.2 feet, B C = .2 feet, and W = 60 lbs., not taking friction into account.

Work applied by P = $\mathbf{P \times 1.2}$.

Work of W = $\mathbf{60 \times .2}$ ;

$\therefore$ $\mathbf{P \times 1.2 = 60 \times .2}$ ;

$\therefore$ P = 10 lbs.

This calculation shows that the advantage of the power depends upon the thinness of the back of the wedge. However, it is necessary to observe, that the astonishing power of the wedge, as usually employed, is equally resolvable into the force of impact, which is considered hereafter.

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## OF THE SCREW.

In this simple machine the pressure applied moves in a circle whose radius is the length of the lever, or arm of the screw, whilst the direction in which the work is done, is a right right.

*Ques.* 26. The lever of a simple screw is 6 feet, the thickness of the thread = .04 feet ; If a pressure of 200 lbs. be applied to the lever, what pressure will be produced on the press-board ?

Space moved over by P, in one revolution,

$= 2 \times 6 \times 3.1416.$

Space moved over by W, in one revolution = .04 feet.

Work of P, in one revolution,

$= \mathbf{2 \times 6 \times 3.1416 \times 200}$ ;

Work of W, in one revolution,

$= \mathbf{W \times .04}.$

$\therefore$ $\mathbf{W \times .04 = 2 \times 6 \times 3.1416 \times 200.}$

$\therefore$ W = 188496 lbs.

The equation obtained above, shows that the efficacy of the screw is obtained by increasing the length of the lever, or by decreasing the thickness of the threads.

*Ques.* 27. The lever of a screw is 4 feet, and the thickness of the threads $\frac{1}{4}$ inch ; required the pressure that must be exerted on the lever to produce a pressure of 12 tons upon the press-board?

In one revolution of the lever, the work done,

$$= 12 \times 2240 \times \frac{1}{48} = 560 ;$$

for $\frac{1}{4}$ of an inch $= \frac{1}{48}$ of a foot.

In one revolution of the lever the work applied,

$$= P \times 8 \times 3.1416 ;$$

$$\therefore \quad P \times 8 \times 3.1416 = 560,$$

$$\therefore \quad P = 22.28 \text{ lbs.}$$

*Ques.* 28. The lever of a simple screw is 2 feet, what must be the thickness of the threads, so that a pressure of 5000 lbs. may be produced on the press-boards, by a pressure of 100 lbs. on the handle, or lever?

Let $x$ be the thickness of the thread, then, the work applied in one revolution,

$$= 4 \times 3.1416 \times 100 = 1256.64 ;$$

Work done,

$$= x \times 5000 = 1256.64 ;$$

$$\therefore \quad x = .2513 \text{ feet.}$$

---

## OF THE COMPOUND SCREW.

THIS mechanical power consists of two screws ; one of which screws within the other, so that whilst the large one is descending, the small one is relatively rising within the large one. In consequence of this compound motion, one revolution of the lever, causes the press-board only to descend a distance equal to the difference of the thickness of

the threads of the screws. Hence the advantage depends upon the smallness of this difference, and not upon the absolute size of the threads; and hence the machine becomes analogous to the compound wheel and axle.

*Ques.*29 . In a compound screw, the length of the lever is 2.5 feet, the distance between the threads of the large or hollow screw is $\frac{2}{3}$ of an inch, and of the small one half an inch; if 60 lbs. pressure be applied to the lever, what is the pressure on the press-board?

In one revolution the large screw descends $\frac{2}{3}$ of a inch, but at the same time, the small screw, by turning within the large one, ascends $\frac{1}{2}$ an inch; therefore the press-board must descend a space =

$$\frac{2}{3} - \frac{1}{2} = \frac{1}{6} \text{ of an inch} = \frac{1}{72} \text{ of a foot.}$$

Work done in one revolution,

$$= \mathbf{W} \times \frac{1}{72};$$

Work applied in one revolution,

$$\mathbf{60 \times 2 \times 2.5 \times 3.1416 = 942.48};$$

$$\therefore \quad \mathbf{W} \times \frac{1}{72} = \mathbf{942.48}.$$

$$\mathrm{W} = 67858.56.$$

*Ques.* 30. If the length of the lever = $3\frac{1}{2}$ feet, the thickness of the thread of the larger screw = $\frac{3}{4}$ of a inch, what must be the thickness of the thread of the smaller screw, so that 20 lbs. applied to the lever, may produce a pressure of 20 tons = 44800 lbs?

$$\frac{3}{4} \text{ of an inch} = \frac{1}{16} \text{ of a foot.}$$

Let $x$ be the thickness of the smaller screw in feet.

$$\therefore \quad \mathbf{20 \times 7 \times 3.1416 = 44800} \left(\tfrac{1}{16} - x.\right)$$

$$\therefore \quad \tfrac{1}{16} - x = .0098214;$$

$$\therefore \quad .0723214 = x.$$

## OF THE ENDLESS SCREW.

In this machine, the threads of a screw cut upon a cylinder, are made to act upon the teeth of a cog-wheel having an axle, round which a cord coils, as in the common windlass.

This combination gives a very slow motion to the weight

*Ques.* 31. In an endless screw, the length of the handle is 3 feet, the number of teeth in the cog-wheel is 24, and the radius of the axle $\frac{3}{4}$ of a foot: if 36 lbs. pressure be applied to the handle, what weight will be raised, friction being neglected?

Since the screw is fixed upon the axis of the handle, one turn will cause one of the teeth of the wheel to be moved round; consequently the handle must make 36 turns, whilst the cog-wheel and axle make one turn.

Work done in one revolution of the axle,

$$= W \times \tfrac{3}{4} \times 2 \times 3.1416;$$

for W will be raised 4.7124 feet.

The work applied in one revolution of the axle,

$$= 36 \times 6 \times 3.1416 \times 24;$$

$$\therefore W \times \tfrac{3}{4} \times 2 \times 3.1416 = 36 \times 6 \times 3.1416 \times 24;$$

$$\therefore W = \frac{36 \times 6 \times 24}{2 \times \frac{3}{4}}$$

$$W = 3456 \text{ lbs.}$$

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## THE HYDROSTATIC PRESS.

This powerful machine consists of two cylinders of different sizes, each of which has a piston, and pressure is transmitted from the small piston to the large one, by means of water. The pressure of the power is applied to the small piston by a lever of the second kind; and the

advantage of the machine depends upon the length of this lever, and the extent of surface of the large piston, compared with that of the small one.

*Ques.* 32. In a hydrostatic press, the surface of the pistons are 3 and 350 square inches respectively, the lever is 30 inches long, and the piston rod is attached 2 inches from the fulcrum ; If a pressure of 100 lbs. be applied to the lever, what pressure will be produced upon the large piston ?

Let the small piston descend $\frac{1}{3}$ of an inch, or $\frac{1}{36}$ part of a foot, then 1 cubic inch of water will be thrown into the large cylinder, and its piston will be raised $\frac{1}{350}$ part of an inch, or $\frac{1}{4200}$ of a foot ;

Work on the small piston,

$$\frac{100 \times 30}{2} \times \frac{1}{36} = 41\frac{2}{3}$$

Work on the large piston,

$$W \times \frac{1}{4200};$$

$$\therefore \quad \frac{W}{4200} = 41\frac{2}{3}$$

$$W = 175000 \text{ lbs.}$$

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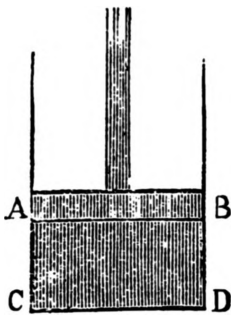
## CHAPTER V.

### WORK OF STEAM AND THE STEAM ENGINE.

If steam in the cylinder A D exert a constant, or mean, pressure upon the piston A B, say of 25 lbs. per square inch, then if a weight of 25 lbs. be placed upon every square inch of surface in the piston, the steam would just be able to move the piston with its weights or pressures through the length of the stroke in opposition to the weight or pressure ; therefore the work performed upon 1 square




inch of the piston in one stroke, will be the pressure of the steam upon 1 inch multiplied by the number of feet in the stroke, and the work upon the whole piston will be the work upon 1 inch multiplied by the number of square inches of the whole piston. In the high pressure engine, the pressure of the atmosphere, about 15 lbs. to the square inch, is opposed to the pressure of the steam. Besides this, a considerable portion of the pressure of the steam is required to overcome the friction of the parts of the engine. As a mean estimate 1 lb. to the square inch is allowed for the friction due to the engine when unloaded; and an additional friction of $\frac{1}{7}$ the effective pressure, or *useful load*, for the resistance necessary to overcome the friction of the loaded engine. Thus, if the pressure of the steam = 56 lbs. on the square inch, from which, if 15 lbs. be taken for the pressure of the air, and 1 lb. for the resistance of the friction of the unloaded piston, then the remainder 40 lbs. will be taken up by the useful load, and the friction arising from that load, that is

$$\text{load} + \frac{\text{load}}{7} = 40 \text{ lbs}$$

$$\text{or } \frac{8 \text{ load}}{7} = 40 \text{ lbs.}$$

$$\therefore \text{ load} = 35 \text{ lbs.}$$

This load is termed the effective pressure of the steam.

In the condensing engine, the pressure of the vapor in the condenser, must be taken in the place of the atmospheric pressure, estimated at a maximum, the pressure of the vapor in the condenser is about 4 lbs. per square inch; then

$$\text{load} + \frac{\text{load}}{7} \times + 4 = \text{total pressure of the steam.}$$

In a condensing engine, let the total pressure of the steam = 61 lbs. per square inch,

$$\therefore \frac{8 \text{ load}}{7} = 65 - 5;$$

$$\therefore \text{ load} = 49 \text{ lbs;}$$

that is, the effective pressure of the steam, in this case = 49 lbs.

EXAMPLES.

*Question* 1. THE area of the piston of a steam engine = 2200 square inches, the mean effective pressure of the steam 15 lbs. per square inch, the length of the stroke 8 feet, the number of strokes per minute = 20; what is the horse power of the engine?

Work done upon 1 square inch of the piston in one stroke,

$$= 15 \times 8 = 120.$$

Work upon the whole piston in one stroke,

$$= 120 \times 2200 = 264000$$

Work done in one minute, or in 20 strokes,

$$= 264000 \times 20 = 5280000'.$$

But the power of a horse being,

$$33000'.$$

$$\therefore \frac{5280000'}{33000'} = 160^{\text{h.}}$$

*Ques.* 2. The area of the piston of a high pressure engine is 625 square inches, the length of the stroke 6 feet, the pressure of the steam 40 lbs. per square inch, the number of strokes per minute 20; it is required to find the number of cubic feet of water which the engine will pump from a mine whose depth is 396 feet, making the usual allowance for friction and the modulus of the pump.

The *modulus* of a machine, is the fraction which expresses the relation of the work done to the work applied. For example, if a machine should only perform one-half the work that is applied to it, then the *modulus* in this case, would be $\frac{1}{2} = .5$. However perfectly a machine may be constructed, there must always be a certain amount of work destroyed by friction. The work that is thus destroyed depends upon the extent and nature of the rubbing surfaces. In many machines also, the power of the laboring agent may be exerted in a more or less efficient manner. Hence it is, that the *modulus* of one machine sometimes differs very much from that of another. The following table of

machines for the raising of water is taken from *Morin's Mechanique Pratique.*

| | Modulus. |
|---|---|
| Incline Chain pump | .38 |
| Upright Chain pump | .53 |
| Bucket Wheel | .60 |
| Chinese Wheel | .58 |
| Archimedian Screw | .70 |
| Pumps for draining mines | .66 |

$$\text{Load} + \tfrac{1}{7}\text{ load} + 1 + 15 = 40;$$

$$\frac{8 \text{ load}}{7} = 24;$$

$$\text{load} = 21 \text{ lbs.}$$

Useful work of the engine per minute,

$$= \mathbf{21 \times 625 \times 6 \times 20 \times .66 = 1039500'}.$$

Work to pump one cubic feet of water a height of 396 feet,

$$= \mathbf{62.5 \times 396 = 24750},$$

Consequently the number of cubic feet that will be pumped per minute,

$$= \frac{\mathbf{1039500'}}{\mathbf{24750}} = 42 \text{ cubic feet.}$$

*Ques.* 3. The area of the piston of a high pressure engine = 660 square inches, the length of stroke = 6 feet, the number of strokes per minute = 20; what must be the mean effective pressure of the steam, so that the engine may do the work of 25 horses?

Let $x$ be the number of lbs. pressure on each square inch of the piston, then, the work per minute,

$$= x \times \mathbf{660 \times 6 \times 20};$$

$$\therefore \quad x \times \mathbf{660 \times 6 \times 20 = 25 \times 33000};$$

$$\therefore \quad x = \frac{\mathbf{825000}}{\mathbf{79200}} = 10\tfrac{5}{12} \text{ lbs.}$$

7) 10.4166
1.4880
11.9046
15.0000
1.0000
27.9046 lbs. total pressure of the

*Ques.* 4. The area of a piston of a high pressure engine = 3000 square inches, the length of stroke = 11 feet, the number of strokes per minute = 16; required the mean pressure of the steam, so that the engine may perform the work of 224 horses, making the usual allowance for friction.

Let $x$ lbs. be the effective pressure of the steam.

Work per minute with $x$ lbs. effective pressure,

$= x \times 3000 \times 11 \times 16 = 528000 \times x$;

The effective work per minute will also be expressed by

$$33000 \times 224 = 7392000'.$$

$$\therefore\ 528000 \times x = 7392000'.$$

$$\therefore\ x = 14 \text{ lbs. effective pressure.}$$

$1 + 15 + 14 + \frac{1}{7}$ of $14 = 32$ lbs. mean pressure of steam.

*Ques.* 5. The length of the stroke of a high pressure engine = 8 feet, the area of the piston 1000 inches, and the number of strokes per minute = 20; what must be the pressure of the steam so that the engine may pump 66 cubic feet of water per minute from a mine whose depth is 896 feet, making the usual allowances for friction and modulus of the pump?

The work done in one minute,

$= 66 \times 62.5 \times 896 \times 3696000'$;

The modulus of the pump = .66;

Hence the work of the engine per minute

$\times .66 = 369000'$, therefore the work per minute,

$$= \frac{3696000'}{.66} = 56000'.$$

The useful work done one inch of the piston in one stroke,

$$= \frac{5600000'}{1000 \times 20} = 280.$$

$$\text{Useful load} = \frac{280}{8} = 35 \text{ lbs.};$$

consequently the pressure of the steam

$$= 35 + \frac{35}{7} + 15 + 1 = 56 \text{ lbs.}$$

In the steam engine, the true source of work is the evaporating power of the boiler.

The magnitude of the work not only depends upon the quantity of water evaporated in a given time, but also upon the temperature, and consequently the pressure at which the steam is formed. Experimental tables have been framed, giving the relation of the volume and pressure of steam from a cubic foot of water; *from these* tables may be found the volume of steam when its pressure and volume of water are given, and *vice versa*. The following will serve as a specimen of such tables.

*Volume of a cubic foot of water in the form of steam at the corresponding pressures, and Temperatures.*

| *Total pressure in pounds, per square inch.* | *Corresponding temperature, by Fahrenheit's thermometer.* | *Volume of the Steam compared with the volume of the water that has produced it.* | *Total pressure in pounds per sq. inch.* | *Corresponding temperature, by Fahrenheit's thermometer.* | *Volume of Steam; Volume of water = 1.* |
|---|---|---|---|---|---|
| 1 | 102.9 | 20954 | 11 | 197.0 | ·2222 |
| 2 | 126.1 | 10907 | 12 | 201.3 | 2050 |
| 3 | 141.0 | 7455 | 13 | 205.3 | 1903 |
| 4 | 152.3 | 5695 | 14 | 209.0 | 1777 |
| 5 | 161.4 | 4624 | 15 | 213.0 | 1669 |
| 6 | 169.2 | 3901 | 16 | 216.4 | 1572 |
| 7 | 176.0 | 3380 | 17 | 219.6 | 1487 |
| 8 | 182.0 | 2985 | 18 | 222.6 | 1410 |
| 9 | 187.4 | 2676 | 19 | 225.6 | 1342 |
| 10 | 192.4 | 2427 | 20 | 228.3 | 1280 |

| *Total pressure in pounds, per square inch.* | *Corresponding temperature, by Fahrenheit's thermometer.* | *Volume of the Steam compared with the volume of the water that has produced it.* | *Total pressure in pounds per sq. inch.* | *Corresponding temperature, by Fahrenheit's thermometer.* | *Volume of Steam; Volume of water = 1.* |
|---|---|---|---|---|---|
| 21 | 231.0 | 1224 | 61 | 294.9 | 460 |
| 22 | 233.6 | 1172 | 62 | 295.9 | 453 |
| 23 | 236.1 | 1125 | 63 | 297.0 | 447 |
| 24 | 238.4 | 1082 | 64 | 298.1 | 440 |
| 25 | 240·7 | 1042 | 65 | 299.1 | 434 |
| 26 | 243.0 | 1005 | 66 | 300.1 | 428 |
| 27 | 245.1 | 971 | 67 | 301.2 | 422 |
| 28 | 247.2 | 939 | 68 | 302.2 | 417 |
| 29 | 249.2 | 909 | 69 | 303.2 | 411 |
| 30 | 251.2 | 882 | 70 | 304.2 | 406 |
| 31 | 253.1 | 855 | 71 | 305.1 | 401 |
| 32 | 255.0 | 831 | 72 | 306.1 | 396 |
| 33 | 256.8 | 808 | 73 | 307.1 | 391 |
| 34 | 258.6 | 786 | 74 | 308.0 | 386 |
| 35 | 260.3 | 765 | 75 | 308.9 | 381 |
| 36 | 262.0 | 746 | 76 | 309.9 | 377 |
| 37 | 263.7 | 727 | 77 | 310.8 | 372 |
| 38 | 265.3 | 710 | 78 | 311.7 | 368 |
| 39 | 266.9 | 693 | 79 | 312.6 | 364 |
| 40 | 268.4 | 677 | 80 | 313.5 | 359 |
| 41 | 269.9 | 662 | 81 | 314.3 | 355 |
| 42 | 271.4 | 647 | 82 | 315.2 | 351 |
| 43 | 272.9 | 634 | 83 | 316.1 | 348 |
| 44 | 274.3 | 620 | 84 | 316.9 | 344 |
| 45 | 275.7 | 608 | 85 | 317.8 | 340 |
| 46 | 277.1 | 596 | 86 | 318.6 | 337 |
| 47 | 278.4 | 584 | 87 | 319.4 | 333 |
| 48 | 279.7 | 573 | 88 | 320.3 | 330 |
| 49 | 281.0 | 562 | 89 | 321.1 | 326 |
| 50 | 282.3 | 552 | 90 | 321.9 | 323 |
| 51 | 283.6 | 542 | 91 | 322.7 | 320 |
| 52 | 284.8 | 532 | 92 | 323.5 | 317 |
| 53 | 286.0 | 523 | 93 | 324.3 | 313 |
| 54 | 287.2 | 514 | 94 | 325.0 | 310 |
| 55 | 288.4 | 506 | 95 | 325.8 | 307 |
| 56 | 289.6 | 498 | 96 | 326.6 | 305 |
| 57 | 290.7 | 490 | 97 | 327.3 | 302 |
| 58 | 291.9 | 482 | 98 | 328.1 | 299 |
| 59 | 293.0 | 474 | 99 | 328.8 | 296 |
| 60 | 294.1 | 467 | 100 | 329.6 | 293 |

*Ques.* 6. In a high pressure engine, the area of the piston = 120 inches, the length of the stroke = 2.4 feet, the effective evaporation of the boiler = .5 cubic feet per minute, the pressure of the steam in the cylinder 64 lbs. per square inch; making the usual allowance for the loss due to friction, required the useful load per square inch of piston, and the useful horse power?

$$\tfrac{8}{7} \text{ useful load} = 64 - 15 - 1 = 48.$$

$$\text{Useful load} = 42 \text{ lbs.}$$

From the table it will be found that a cubic foot of water raised to steam of 64 lbs. pressure, has a volume of 440 cubic feet.

Volume of steam evaporated per minute

$$= 440 \times .5 = 220 \text{ cubic feet;}$$

Volume discharged at each stroke

$$= \frac{120 \times 2.4}{144} = 2 \text{ cubic feet;}$$

The number of strokes each minute

$$= \frac{220}{2} = 110.$$

The work in one stroke

$$= 42 \times 120 \times 2.4 = 12096.$$

Work in one minute

$$= 12096 \times 110 = 1330560'.$$

$$\text{Horse power} = \frac{1330560'}{33000'} = 40^{\text{h.}}\,32.$$

*Ques.* 7. The area of the piston of a high pressure engine is 264 inches, the length of stroke $5\frac{1}{2}$ feet, the pressure of the steam in the cylinder = 72 lbs. per square inch, the number of strokes per minute = 36; required the useful load, the water evaporated per hour, and the useful horse power of the engine?

Let $x$ be the useful load, then

$$x + \frac{x}{7} + 15 + 1 = 72 \text{ lbs.}$$

$$7x + x = 392.$$

$$x = 49 \text{ lbs.}$$

The volume of steam discharged per minute, in cubic feet

$$= \frac{264 \times 5.5}{144} \times 36 = 363 \text{ cubic feet.}$$

The cubic feet of steam discharged in an hour

$$= 363 \times 60 = 21780.$$

From the table it will be found that one cubic foot of water yields 396 cubic feet of steam at 72 lbs. pressure.

Consequently the cubic feet of water evaporated per hour

$$= \frac{21780}{396} = 55.$$

The useful horse power of the engine

$$= \frac{\mathbf{264 \times 49 \times 5.5 \times 36}}{\mathbf{33000}} = \mathbf{77^{h.}\ 616.}$$

It has been found by experiment, that whatever may be the pressure at which steam is formed, the quantity of fuel necessary to evaporate a given volume of water, is nearly the same. Hence it follows, that it is most advantageous to employ steam of a high pressure.

*Ques.* 8. Required the duty of the engine, in the last example, allowing a bushel of coals 94 lbs., to evaporate 11.5 cubic feet of water?

The useful work per hour

$$= \mathbf{264 \times 49 \times 5\tfrac{1}{2} \times 36 \times 60 = 153679680};$$

which is the work of 55 cubic feet of water, or of 1 bushel of coal.

$$= \mathbf{153679680 \times \frac{11.5}{55} = 32133024},$$

which is termed the duty of the engine.

*Ques.* 9. A train of 200 tons, moves along at a uniform speed of 30 miles an hour upon a level rail, the resistance of friction upon the rail is $5\frac{1}{2}$ lbs. per ton, the resistance of the atmosphere 33 lbs. upon the whole train when the speed is 10 miles an hour, the diameter of the driving wheel 7 feet, the area of the piston 120 inches, the length of the stroke $= \frac{3}{2}$ feet, and in addition to the resistances of fric-

tion, the resistance due to the blast pipe is $1\frac{3}{4}$ lbs. per sq. inch of the piston, when the speed of the train is 10 miles an hour. Required the pressure of the steam, the evaporation of the boiler, and the number of bushels of coal necessary for a journey of 500 miles, supposing that 1 bushel, or 94 lbs. will evaporate $11\frac{1}{2}$ cubic feet of water?

When the diameter of a circle = 7 feet, the circumference = 22 feet nearly, which is the space passed over in one revolution of the driving wheel.

Total resistance to the motion of the carriages

$$= 200 \times 5.5 + \left(\frac{30}{10}\right)^2 \times 33 = 1397 \text{ lbs.}.$$

The work in one revolution

$$= \mathbf{1397 \times 22 = 30734.}$$

Work of 1 lb. of pressure per square inch of the piston in one revolution

$$= \mathbf{1 \times 120 \times \frac{3}{2} \times 4 = 720,}$$

for the engine has two cylinders, and each piston makes two strokes while the driving wheel turns once round.

Consequently, the effective pressure on one square inch of the piston,

$$= \frac{\mathbf{30734}}{\mathbf{720}} = 42.78 \text{ lbs.}$$

It has been found by experiment, that the resistance of the blast pipe increases with the speed of the engine,

$\therefore$ Resistance due to the blast pipe

$$= 1\frac{3}{4} \times \frac{30}{10} = 4\frac{1}{4} \text{ lbs.}$$

Hence the total pressure of the steam on the piston

$$= 42.78 + \frac{42.78}{7} + 1 + 15 + 4\frac{1}{4} = 69.14 \text{ lbs.}$$

The number of revolutions of the driving wheel per minute

$$= \frac{30 \times 5280}{60 \times 22} = 120;$$

therefore, the two pistons will make $120 \times 4 = 480$ strokes a minute.

The cubic feet of steam discharged per minute will be

$$= \frac{120}{144} \times \frac{3}{2} \times 480 = 600.$$

From the table it will be found that a cubic foot of water produces 411 cubic feet of steam of 69.14 lbs. pressure.

The number of cubic feet of water evaporated per minute

$$= \frac{600}{411}.$$

As one bushel of coal evaporates 11.5 cubic feet of water, the number of bushels of coal used per minute

$$= \frac{600}{411 \times 11.5};$$

A journey of 500 miles is performed in $16\frac{2}{3}$ hours, or in 1000 minutes, at the rate of 30 miles an hour.

Therefore the number of bushels of coal for 1000 minutes

$$= \frac{600 \times 1000}{411 \times 11.5} = 126.95 \text{ bushels.}$$

*Ques.* 10. In a locomotive engine, the area of the piston is 90 inches, the length of the stroke 16 inches, the pressure of the steam 50 lbs., the effective evaporation of the boiler .7 cubic feet per minute, the diameter of the driving wheel 5 feet; what is the speed of the train per hour?

At 50 lbs pressure, 1 foot of water forms 554 cubic feet of steam.

The volume of steam generated per minute

$$= 554 \times .7 = 387.8.$$

Cubic feet discharged in one revolution of the driving wheel

$$= 4 \times \frac{90 \times 16}{1728} = 3\frac{1}{3}.$$

Therefore the number of revolutions of the driving wheel per minute

$$= \frac{387.8}{3.33} = 116.3,$$

$\therefore$ The space moved over by the carriage per minute, in feet

$$= 5 \times 3.1416 \times 116.3.$$

Consequently the miles moved over per hour

$$= \frac{5 \times 3.1416 \times 116.3 \times 60}{5280} = 20.7.$$

*Ques.* 11. The area of the piston of a locomotive engine = 80 square miles, the length of the stroke = 15 miles, the pressure of the steam 48 lbs. per square inch, and the diameter of the driving wheel 5 feet; required the effective evaporation of the boiler, so that the train may have a speed of 30 inch. an hour; required also the effective horse power of the engine, and the weight of the train, taking the resistances to the motion of the piston as in former examples?

Number of revolutions of the wheel per minute

$$= \frac{30 \times 5280}{60 \times 5 \times 3.1416} = 168.$$

Number of strokes of both pistons per minute

$$= 168 \times 4 = 672.$$

$$\text{Load} + \frac{\text{load}}{7} + 15 + 1 + \frac{30}{10} \times 1.75 = 48 \text{ lbs.}$$

$$\text{load} = \frac{26.75 \times 7}{8} = 23.4 \text{ lbs.}$$

The effective work per minute will be

$$= \mathbf{23.4 \times 80 \times \tfrac{5}{4} \times 672 = 1572480'},$$

Hence, the effective horse power

$$= \frac{\mathbf{1572480'}}{\mathbf{33000'}} = \mathbf{47}^{\text{h.}}$$

Now the effective work has to support a speed of 30 miles an hour, 2640 feet per minute, in the train in opposition to the resistances of friction and the atmosphere.

Work due to the resistance of the atmosphere per minute

$$= \left(\frac{\mathbf{30}}{\mathbf{10}}\right)^2 \times \mathbf{33} \times \mathbf{2640} = \mathbf{784080'}.$$

Therefore the work due to friction per minute

$$= 1572480' - 784080' = 788400'.$$

Work of friction when the train weighs $x$ tons

$$= 7 \times 2640 \times x;$$

$$\therefore \quad x = \frac{788400'}{7 \times 2640} = 42.6 \text{ tons.}$$

Cubic feet of steam discharged each stroke

$$= \frac{90 \times 1.25}{144};$$

the cubic feet discharged in a minute

$$= \frac{80 \times 1.25 \times 672}{144}$$

From the table it will be found, that a cubic foot of water yields 573 cubic feet of steam, therefore, the number of cubic feet of water evaporated per minute

$$= \frac{80 \times 1.25 \times 672}{144 \times 573} = 81 \text{ cubic feet.}$$

*Question* 12. A railroad train of 60 tons. ascends an incline that has a rise of $\frac{1}{4}$ in 100; required the maximum speed per hour, 40 being the effective horse power of the engine, friction, 8 lbs. per ton?

Let $x =$ the speed of the train in feet per hour.

$$100 \ : \ \tfrac{1}{4} \ :: \ x \ : \ \frac{x}{400} =$$

the rise of the rail in $x$ feet.

Work due to gravity

$$= \frac{60 \times 2240 \times x}{400} = 336\,x.$$

Work of friction, $= 60 \times 8 \times x = 480\,x;$

But the work due to gravity per hour, added to the work due to friction per hour, must be equal to the work done by the engine in the same time;

$$\therefore \ 816\,x = 40 \times 33000 \times 60 = 79200000.$$

$$\therefore \quad x = \frac{79200000}{816} = 97056 \text{ ft.} = 18.4 \text{ miles.}$$

If the engine in the last example move this train, what must be the effective evaporation of the boiler, and the duty of the engine.

Speed per minute, $= \frac{97056}{60} = 1617.6$ feet.

Number of strokes of the pistons per minute,

$$= \frac{1617.6}{5 \times 3.1416} \times 4 = 412;$$

The effective work of the engine per minute,

$$= 33000 \times 40 = 1320000'.$$

Suppose $y$ = the pounds effective pressure on each square inch of the piston, then the work of $y$ lbs. effective pressure per minute

$$y \times 80 \times \tfrac{15}{12} \times 412 = 1320000'.$$

$$\therefore \quad y = 32 \text{ lbs.}$$

Pressure of steam

$$= 32 + \tfrac{1}{7} \times 32 + 15 + 1 + \frac{18.4}{10} \times 1.75 = 55.7 \text{ lbs.}$$

One cubic foot of water in the form of steam, at 55.7 lbs. pressure, is 504 cubic feet. Number of cubic feet discharged per minute, in the form of steam

$$= \frac{80 \times 15}{1728} \times 412 = 287.$$

$\therefore$ Number of cubic feet of water evaporated per minute

$$= \frac{287}{504} = .57.$$

Now this water performs $1320000'$,

$\therefore$ 11.5 cubic feet of water, or 94 lbs. of coal

$$= \frac{11.5}{.57} \times 1320000' = 26654077,$$

the duty of the engine.

## OF THE WORK DEVELOPED BY THE CONDENSATION OF STEAM.

When water is raised into steam at the boiling point or temperature, of 212° Fahr., its volume is increased 1711 times, or a cubic inch of water will very nearly form a cubic foot of steam. Now, if steam at this temperature be allowed to enter the lower part of the cylinder, (see the last figure), then the pressure beneath the piston will just counterpoise the pressure of the air upon the piston, and a small additional force will cause the piston to rise. If, then, the steam be condensed by a jet of cold water, a vacuum will be formed, and the piston will be pressed downward with the whole weight of the atmosphere resting upon the surface of the piston. But, it has been found, that a perfect vacuum cannot be formed in this way, because water gives off vapor at all temperatures. Now, if 14.7 lbs. be taken as the mean pressure of the atmosphere, upon one square inch of surface, then by the condensation of the steam, upon an average, an effective pressure of

$$14.7 - 4 = 10.7 \text{ lbs.}$$

is established upon every square inch of the piston ; taking the temperature of the condenser at 150°, at which the corresponding pressure of the vapor is 4 lbs. per square inch. However, when the temperature of the condenser is given, the pressure of the vapor is easily found.

### EXAMPLES.

*Ques.* 13. Required the work developed, by the condensation of a cubic foot of water, supposing 4 lbs. to be the elasticity of the vapor after condensation ; required also, the duty of the atmospheric engine using the steam in this manner.

Cubic feet of vacuum formed by condensation.

$$= 1710 = 1711 - 1.$$

Pressure on one square inch of the piston

$$= 14.7 - 4 = 10.7 \text{ lbs.}$$

Suppose the area of the piston to be one square foot, the length of the stroke will be 1710 feet.

Consequently, the work of one cubic foot of water

$$= 144 \times 10.7 \times 1710 = 2634768.$$

And the duty or work of 94 lbs. of coal = 1 bushel, = duty of $11\frac{1}{2}$ cubic feet of water

$$= 2634768 \times 11.5 = 302998320.$$

*Ques.* 14. What must be the effective evaporation of the boiler of an atmospheric engine, so that the horse power may be 30, allowing 3 lbs. for the elasticity of the vapor in the condenser.

Work of the engine per minute

$$30 \times 33000' = 990000'.$$

Work of one cubic foot of water

$$= (14.7 - 3) \times 144 \times 1710 = 2881008.$$

Therefore, the uumber of cubic feet of water evaporated per minute

$$= \frac{990000'}{2881008} = .34.$$

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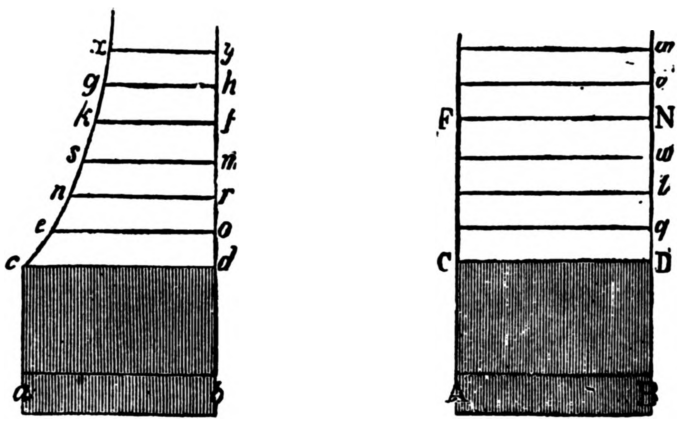
## OF THE USE OF STEAM WHEN USED EXPANSIVELY.

WHEN steam is used expansively, it is allowed to enter the cylinder for only a part of the stroke, and then, for the remaining portion, the piston is moved by the expansive force of the steam. This is the most economical way of employing steam power; for all the available work is taken out of the elastic vapor before it is condensed.

When the volume of steam is increased, its elasticity or pressure is decreased in the same ratio; that is, if its volume is increased three times, its pressure will be one-third of what it was at first, and so on. This is Boyle's law, but M. Regnault has shown, that it will not hold in extreme cases.

Let the steam be cut off when the piston is at C D, and let the remaining part of the stroke be divided into any even number of parts ; then the pressure of the steam upon the piston when it arrives at the different lines, forming the divisions, may be ascertained by the law just explained.




Let *d e*, *e o*, *r n*, &c., be lines containing as many units as there are units of pressure on the piston at the corresponding points of the stroke, then the work done from A B to C D, will be the area of the rectangle *a d*, because the work is the product of the pressure *c d*, by the space *d b ;* the work done from C D to F N, will be represented by the units in the curved space *c d f k*, because the work done from D to *q* is very nearly the pressure *c d* multiplied by the space *d o*, and the work done from *q* to *t*, is very nearly the pressure *o e* multiplied by *o r*, and so on. Now the smaller these intervals are taken, the more nearly will the areas represent the work. In order to find the area of this space, take the following Rule :—To the sum of the extreme ordinates, add four times the sum of the even ordinates, and twice the sum of the odd ordinates ; then this sum multiplied by one-third the common distance between the ordinates will give the area of *c k f d.*

### EXAMPLES.

*Ques.* 15. In a condensing engine the length of the stroke is 5 feet, the steam is cut off at 2 feet of the stroke, the pressure of the steam in the cylinder 48 lbs., and the elasticity of the vapor in the condenser is 4 lbs. ; required the

work performed upon one square inch of the piston in one stroke?

Let the space through which the steam acts expansively, be divided into six equal parts, then each interval will be $\frac{1}{2}$ a foot, by Boyle's law.

$$c\,d = 48 = 48.\ \text{lbs.}$$

$$o\,e = \frac{2 \times 48}{2\frac{1}{2}} = 38.4\ \text{lbs.}$$

$$n\,r = \frac{2 \times 48}{3} = 32.\ \text{lbs.}$$

$$s\,m = \frac{2 \times 48}{3\frac{1}{2}} = 27.4\ \text{lbs.}$$

$$k\,f = \frac{2 \times 48}{4} = 24.\ \text{lbs.}$$

$$g\,h = \frac{2 \times 48}{4\frac{1}{2}} = 21.3\ \text{lbs.}$$

$$x\,y = \frac{2 \times 48}{5} = 19.2\ \text{lbs.}$$

Area of $c\,d\,y\,x$, or work done expansively on 1 square inch of the piston in one stroke

$$= \frac{\frac{1}{2}}{3}\left\{48 + 19.2 + 4\,(38.4 + 27.4 + 21.3) + 2\,(32 + 24)\right\} = 87.9;$$

The work done before the steam is cut off, that is, from A B to C D,

$$= 48 \times 2 = 96;$$

Work done against the piston by the vapor in the condenser

$$= 4 \times 5 = 20.$$

Consequently, the total work on one inch of the piston in one stroke

$$= 87.9 + 96 - 20 = 163.9.$$

*Ques.* 16. The area of a piston of a condensing engine is 1440 square inches, the length of the stroke, including the clearance, is 5 feet, the steam is cut off at 1 foot of the cylinder, the clearance = $\frac{1}{4}$ foot, the pressure of the steam = 30 lbs., the elasticity of the vapor in the condenser = 4 lbs., the effective evaporation of the boiler .2 cubic feet per minute, and the resistances as before described; required the useful load, and the useful horse power of the engine?

By dividing the space through which the steam acts expansively into four equal parts, and calculating the pressures by Boyle's law, the work done expansively on one inch =

$$\frac{1}{3}\left\{30+6+\left(15+7\tfrac{1}{2}\right)\times 4+10\times 2\right\}=48.66.$$

Now the space through which the piston moves before the steam is cut off

$$= 1 - \tfrac{1}{4} = \tfrac{3}{4} \text{ foot};$$

Work done before the steam is cut off

$$= 30 \times \tfrac{3}{4} = 22.5;$$

Total work of steam on 1 square inch in one stroke

$$= 48.66 + 22.5 = 71.16.$$

Mean pressure of the steam $= \dfrac{71.16}{4.75} = 14.9$ lbs.

But the resistances = the mean pressure of the steam,

$$\therefore \text{ Load} + \frac{\text{load}}{7} + 1 + 4 = 14.9.$$

$$\therefore \text{ Load} = 8.66 \text{ lbs.}$$

One cubic foot of water generates 833 cubic feet of steam at 30 lbs. pressure.

$\therefore$ The volume of steam discharged per minute

$$= .2 \times 883 = 176.6.$$

Volume discharged each stroke

$$\frac{1440}{144} \times 1 = 10 \text{ cubic feet.}$$

∴ The number of strokes per minute

$$= \frac{176.6}{10} = 17.66.$$

The useful work per minute will be the continued product of the load, the area of the piston, the length of the stroke, and the number of strokes per minute.

∴ The effective horse power

$$= \frac{8.66 \times 1440 \times 4.75 \times 17.66}{33000} = 31^{h}.7.$$

*Ques.* 17. What must be the evaporation of the boiler of the engine in the last example, when the steam has a pressure of 48 lbs., so that the effective horse power = 40 ?

Proceeding, as in the last example, the load is found to be 16.5 lbs.

Therefore, the effective work in one stroke

$$= 16.5 \times 1440 \times 4.75 = 112860.$$

The effective work in one minute

$$= 40 \times 33000 = 1320000.$$

Therefore, the number of strokes per minute

$$= \frac{1320000}{112860} = 11.69.$$

Since 10 cubic feet of steam is discharged in one stroke, the volume of steam discharged per minute = 10 × 11.69 = 117 cubic feet, nearly. One cubic foot of water forms 575 cubic feet of steam at 48 lbs. pressure.

∴ Cubic feet of water evaporated per minute $= \frac{117}{575}$.

*Ques.* 18. Required the duty of the engine in example 16 ?

Useful work per minute

$$= 31.7 \times 33000.$$

Now, this work is done by the evaporation of .2 feet of water.

$\therefore$ Work of 11.5 feet of water, or 1 bushel of coal.

$$= 31.7 \times 33000 \times \frac{11.5}{.2} = 60 \text{ millions,}$$

units of work, the duty of the engine.

When water, or any body falls from a given height, the work which it is capable of performing, will just be that which would be done upon it in raising it to the height from which it has fallen. It is no matter in what way this work is used ; whether the water falls into the buckets of an overshot wheel, or delivers its work upon the paddles of an undershot wheel, the laboring force of the water will always be equal to the work due to the height of the fall.

*Ques.* 19. The breadth of a stream is 4 feet, depth 3 feet, mean velocity of the water 15 feet per minute. and the height of the fall 20 feet ; required the horse power of the water-wheel which does $\frac{7}{10}$ of the work of the water, and also the number of bushels of corn which the wheel will grind in a day of 10 hours.

Cubic feet of water going over the fall per minute, $= 4 \times 3 \times 15 = 180$ ; weight $= 180 \times 62.5 = 11250$ lbs. As the water descends a height of 20 feet, the work which it is capable to perform

$$= 20 \times 11250 = 225000'.$$

But the wheel does $\frac{7}{10}$ of this work

$$= 225000' \times \frac{7}{10} = 157500'$$

$$\text{Horse power} = \frac{157500'}{33000'} = 4^{\text{h.}}.77 ;$$

But a horse is able to grind a bushel of corn in an hour, therefore, the number of bushels ground in 10 hours

$$= 4.77 \times 10 = 47.7.$$

# CHAPTER VI.

## OF ACCUMULATED WORK.

When a body moves uniformly, the space described is, obviously, equal to the units of time during which the body moves, multiplied by the space passed over in each unit of time; that is to say, the space is equal to the product of the time by the velocity. If, therefore, a rectangle be constructed, having the units in the base equal to the units in the velocity, and the units in the perpendicular equal to the units of time; then the units of surface in the rectangle, will give the units of space moved over by the body. In like manner, the space described by the body, whose motion is uniformly accelerated, may be represented by the area of a trapezoid, whose parallel sides contain respectively the units of velocity at the commencement and end of the motion, and perpendicular between these sides, the units in the time. This proposition will readily be understood, by observing, that the mean length of the trapezoid will contain the same number of units as the velocity, which the body has at the middle of the time; and this will be true, however small the intervals of time may be taken. The best illustration of accelerated motion is afforded in the case of falling bodies.

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## FORCE OF GRAVITY.

When bodies fall freely near the surface of the earth, the force of the earth's attraction being constant, communicates to them equal additions of velocity in equal intervals of time. Thus at the end of one second, the velocity of the body is 32.155 feet; at the end of two seconds, 2 times, 32.155 feet; at the end of 3 seconds, 3 times, 32.155 feet; at the end of 4 seconds, 4 times, 32.155 feet, and so on; or generally, the velocity acquired by a falling body, is equal to the product of the time of the body's fall in seconds by $32\frac{1}{6}$ feet, putting $32\frac{1}{6}$ for 32.155.

This is expressed by the equation,

$$v = t \times 32\tfrac{1}{6}.$$

The space described by a body in one second, will be half of $32\frac{1}{6}$ feet $= 16\frac{1}{12}$ feet; because, the velocity of the body in the middle of the time, will be the mean velocity with which it moves during that time. In like manner, the space described by the body in 4 seconds, will be 4 times $2 \times 32\frac{1}{6}$ feet; because, $4 \times 32\frac{1}{6}$ feet is the velocity at the end of the time, and, therefore, $2 \times 32\frac{1}{6}$ will be the mean velocity, or the velocity in the middle of the time. But 4 times $2 \times 32\frac{1}{6} = 16 \times 16\frac{1}{12} = 4^2 \times 16\frac{1}{12}$, that is, the space described by a falling body in 4 seconds, is equal to the square of the time, multiplied by the space described in one second. In the same manner, any other case may be established. In general, relation of space and time is therefore expressed by

$$S = t^2 \times 16\tfrac{1}{12}.$$

*Ques.* 1. What velocity will a falling body have at the end of 5 seconds?

$$32\tfrac{1}{6} \times 5 = 160\tfrac{5}{6} \text{ feet.}$$

*Ques.* 2. In what time will a body acquire a velocity of 193 feet a second.

$$\frac{193}{32\frac{1}{6}} = 6 \text{ seconds.}$$

*Ques.* 3. Through what space will a body fall in $4\frac{1}{2}$ seconds?

$$(4\tfrac{1}{2})^2 \times 16\tfrac{1}{12} = 325\tfrac{11}{16} \text{ feet.}$$

*Ques.* 4. A body is thrown downward with a velocity of 10 feet per second, how far will it descend in 6 seconds?

It is evident that the body will retain its motion of projection, although acted upon by gravity. Now the space due to the projection $= 6 \times 10 = 60$ feet, and this added to the space due to gravity will give

$$60 \times 6^2 \times 16\tfrac{1}{12} = 253 \text{ feet.}$$

*Ques.* 5. In what time will a body fall 193 feet?

By the general equation

$$t^2 = \frac{193}{16\frac{1}{12}} = 12.$$

$$\therefore \quad t = 3.4 \text{ seconds.}$$

*Ques.* 6. From what height must a body fall, to acquire a velocity of $96\frac{1}{2}$ feet?

First it is necessary to find the time which the body will have to fall, to acquire a velocity of $96\frac{1}{2}$ feet.

$$\frac{96\frac{1}{2}}{16\frac{1}{12}} = 3 \text{ seconds.}$$

$$\text{Height} = 3^2 \times 16\frac{1}{12} = 144\frac{3}{4} \text{ feet.}$$

*Ques.* 7. The velocity of a body is 84 feet. From what height would it have to fall, in order to attain this motion?

$$\text{Time} = \frac{84}{32\frac{1}{6}}\text{; distance or space}$$

$$= \left(\frac{84}{32\frac{1}{6}}\right)^2 \times 16\frac{1}{12} = \frac{84^2}{2 \times 32\frac{1}{6}}.$$

Put $g = 32\frac{1}{6}$ and $a = 84$, then

$$\frac{a^2}{g^2} \times \frac{g}{2} = \frac{a^2}{2g}.$$

$\therefore$ The space fallen through to acquire the velocity $a$, is equal to the square of $(a,)$ divided by $(2\,g)$. In most modern works, $g$ is put for the number $32\frac{1}{6}$, and $\pi$ is put for the circumference of a circle when the diameter $= 1$.

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## OF THE WORK ACCUMULATED BODY.

WHEN a body is in a state of motion, it will continue in that state, unless acted upon by some external force. But, in order to give this motion to the body, there must be

work done upon it. Thus, we may give the velocity of $32\frac{1}{6}$ feet to one pound, by raising it $16_{1\frac{1}{2}}$ feet, and then allowing it to fall by the force of gravity.

In this case, the units of work accumulated in the body will be

$$= \text{☞} \; 16\tfrac{1}{12}$$

Again, when a heavy fly-wheel is in rapid motion, a considerable portion of the work of the engine must have gone to produce this motion; and before the engine can come to a state of rest, all the work accumulated in the fly, as well as in the other parts of the machine, must be destroyed. In this way a fly-wheel acts as a reservoir of work.

IN ORDER TO ESTIMATE THE WORK IN A MOVING BODY, IT IS SIMPLY NECESSARY TO CONSIDER THE HEIGHT FROM WHICH IT MUST FALL TO ACQUIRE THE GIVEN VELOCITY, AND THEN THE WORK WILL BE FOUND, BY MULTIPLYING THAT HEIGHT IN FEET BY THE WEIGHT OF THE BODY IN LBS.; BECAUSE, THE WORK EXPENDED IN RAISING THE BODY, TO THE NECESSARY HEIGHT TO COMMUNICATE THE GIVEN VELOCITY, MUST BE THE SAME AS THE WORK, WHICH GRAVITY WILL PERFORM UPON THE BODY IN ITS DESCENT

*Ques.* 8. The weight of a ram is 600 lbs., and at the end of the blow has a velocity of $32\frac{1}{6}$ feet, what work has been done in raising it?

The ram must have fallen $16_{1\frac{1}{2}}$ feet, the work done upon it therefore

$$= 600 \times 16\tfrac{1}{12} = 9650.$$

*Ques.* 9. How many units of work are accumulated in a body whose weight is 144 lbs., and velocity 200 feet?

The height from which the body must fall, to acquire the given gravity

$$= \frac{200^2}{2 \times 32\frac{1}{6}} = 621.76 \text{ feet.}$$

Consequently, the work which must have been done upon the body

$$= 621.76 \times 144 = 89533.68.$$

*Ques.* 10. Required the work accumulated in a cannon ball, whose weight is $32\frac{1}{6}$ lbs., and velocity of 1500 feet?

The height from which the ball must fall, to acquire a velocity of 1500

$$= \frac{1500^2}{2 \times 32\frac{1}{6}};$$

The work accumulated in the body

$$= \frac{1500^2 \times 32\frac{1}{6}}{2 \times 32\frac{1}{6}} = 1125000.$$

This example clearly shows, that *the work accumulated in a moving body is equal to the square of the velocity in feet per second, multiplied by the weight of the body in lbs., and divided by* $2 \times 32\frac{1}{6}$.

This very important proposition is expressed by the equation

$$U = \frac{V^2 \times W}{2g}.$$

*Ques.* 11. A ball weighing 20 lbs., is projected with a velocity of 60 feet per second, on a bowling green. What space will the ball move over before it comes to rest, allowing the friction to be $\frac{1}{18}$ the weight of the ball?

The units of work, $U$, in the ball

$$= U = \frac{60^2 \times 20}{2 \times 32\frac{1}{6}} = 1119.17.$$

It is evident, that the ball will not stop until all this work is destroyed, that is, until the work destroyed by friction is equal to the accumulated work.

Let $x$ be the number of feet over which the ball moves before it comes to a state of rest, then the work destroyed by friction in moving the ball over $x$ feet

$$= \frac{20}{18} \times x = 1119.17;$$

$$\therefore \quad x = \frac{1119.17 \times 18}{20} = 503.6 \text{ feet.}$$

*Ques.* 12. A train weighs 193 tons, and has a velocity of 30 miles an hour when the steam is turned off; how far will the train move upon a level rail, whose friction is $5\frac{1}{2}$ lbs. per ton?

30 miles an hour = 44 feet per second, for,

$$\frac{30 \times 5280}{60 \times 60} = 44.$$

193 tons = 432320 lbs.

$$\therefore \quad U = \frac{44^2 \times 432320}{2 \times 32\frac{1}{6}} = 13009920.$$

When the train stops, the work of friction will be equal to the work accumulated in the train, hence, if $x$ be the number of feet the train will move after the steam is cut off, the work in moving the train over $x$ feet

$$= 193 \times 5\tfrac{1}{2} \times x = U.$$

$$\therefore 193 \times 5\tfrac{1}{2} \times x = 13009920.$$

$$\therefore \quad x = \frac{13009920}{193 \times 5\frac{1}{2}} = 12256 \text{ feet.}$$

*Ques.* 13. A train weighing 60 tons, has a velocity of 40 miles per hour when the steam is turned off, how far will it ascend an incline of 1 in 100, taking friction at 8 lbs. per ton?

40 miles an hour = $58\frac{2}{3}$ feet a second.

60 tons = 134400 lbs.

Work in the train when the steam is cut off

$$= \frac{(58\frac{2}{3})^2 \times 134400}{2 \times 32\frac{1}{6}};$$

put $x$ = the number of feet the train will move after the steam is turned off; then the work due to the friction

$$= 60 \times 8 \times x = 480\,x;$$

$$100 : 1 :: x : \frac{x}{100} =$$

the rise of the rail in $x$ feet;

$\therefore$ The work due to gravity

$$= \frac{x}{100} \times 134400 = 1344\,x;$$

$$\therefore 480\,x + 1344\,x = \frac{(58\frac{2}{3})^2 \times 134400}{2 \times 32\frac{1}{6}};$$

$$\therefore x = \frac{(58\frac{2}{3})^2 \times 134400}{1824 \times 64\frac{1}{3}} = 3942\tfrac{1}{2} \text{ feet.}$$

*Ques.* 14. Two balls each weighing $64\frac{1}{3}$ lbs., are placed at the extremities of a horizontal arm, which gives motion to a screw driving a punch, as in the common stamping machine. The velocity given to the balls is 8 feet per second. What is the mean resistance opposed to the punch, when it is just driven through an iron plate $\frac{3}{8}$ of an inch thick?

The weight of the two balls $= 128\frac{2}{3}$ lbs.

If the velocity of the balls be 8 feet, just as the punch begins to cut, then the units of work accumulated

$$= U = \frac{8^2 \times 128\frac{2}{3}}{2 \times 32\frac{1}{6}} = 128.$$

Now, if the thickness of the plate be 1 foot, the uniform resistance would obviously be 128 lbs.; but the thickness is $\frac{1}{32}$ of a foot.

$\therefore$ Mean resistance, $\frac{128}{\frac{1}{32}} = 4096$ lbs.

*Ques.* 15. A ball weighs 24 lbs., and is fired from the mouth of a cannon 12 feet long, with the velocity of 1000 feet per second; it is required to find the mean pressure of the elastic vapor upon the ball?

$$U = \frac{1000^2 \times 24}{2 \times 32\frac{1}{6}} = 373057.$$

Let $x$ be the pressure upon the ball during the passage through the barrel, then

$$12 \times x = 373057.$$

$$\therefore \quad x = 31088 \text{ lbs. mean pressure.}$$

*Ques.* 16. A carriage of 1 ton moves on a level rail with the speed of 8 feet per second, through what space must the carriage move, to have a velocity of 2 feet, supposing friction to be 6 lbs. per ton?

$$\text{Work lost} = \frac{8^2 \times 2240}{64\frac{1}{3}} - \frac{2^2 \times 2240}{64\frac{1}{3}} = 2089.$$

But this work has been taken up by friction passing over a space which put $= x$ feet then,

$$x \times 6 = 2089;$$

$$\therefore \quad x = 348 \text{ feet.}$$

*Ques.* 17. If the carriage in the last example move over 400 feet before it comes to a state of rest, what is the resistance of friction per ton?

Work accumulated in the carriage

$$= \frac{8^2 \times 2240}{64\frac{1}{3}} = 2228;$$

Work of friction = friction $\times$ 400;

$$\therefore \quad \text{Friction} \times 400 = 2228;$$

$$\text{Friction} = 5.57 \text{ lbs.}$$

*Ques.* 18. Two weights, W and $w$, weighing 5 and 3 lbs. respectively, are connected by a cord that goes over a fixed pulley, as in Atwood's machine; through what space must W descend to acquire a velocity of 10 feet?

Since the velocity with which $w$ ascends, is the same as the velocity with which W descends, then

$$\text{Work in W} = \frac{10^2 \times 5}{64\frac{1}{3}} = 7.772;$$

$$\text{Work in } w = \frac{10^2 \times 3}{64\frac{1}{3}} = 4.611;$$

$$\text{Total accumulated work} = 12.383.$$

Let $x$ be the space passed over by each of the weights, then

$$\text{Work of gravity on W} = 5 \times x$$

$$\text{Work of gravity on } w = 3 \times x$$

$$2 \times x \text{ difference.}$$

But the work performed upon $w$ has been yielded by the work of W; therefore, the work remaining in the bodies

$$= 2 \times x = 12.383.$$

$$\therefore x = 6.19 \text{ feet.}$$

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## OF THE CENTRE OF GYRATION, AND THE WORK IN A ROTATING BODY.

*Ques.* 19. Two balls, A and B, are connected by a rod, which is made to revolve upon a centre C; the weight of

A is 3 lbs., and that of B is 4 lbs., the distance of A from the axis is 8 feet, and that of B is 5 feet; if a point in the

rod, at 1 foot from the axis C, has a velocity of 10 feet per second ; it is required to determine the work in the balls, and the point in the rod where we may suppose the weight of the two balls collected, so that the work may not be altered ?

Velocity $\mathbf{A} = 8 \times 10$ ; Velocity of $\mathbf{B} = 5 \times 10$ ;

$$\text{Work in } \mathbf{A} = \frac{8^2 \times 10^2 \times 3}{64\frac{1}{2}} = 298.44$$

$$\text{Work in } \mathbf{B} = \frac{5^2 \times 10^2 \times 4}{64\frac{1}{3}} = 155.44$$

$$\text{Total work} = 453.88$$

Let $x$ be the distance from the axis $=$ C $n$, then the work of the two balls collected at $n$

$$= \frac{(10\,x)^2 \times 7}{64\frac{1}{2}} = 453.88 = \frac{80^2 \times 3}{64\frac{1}{2}} \times \frac{50^2 \times 4}{64\frac{1}{3}}$$

$$\therefore \quad 700\,x^2 = 29200$$

$$x^2 = 41.71$$

$$\therefore \quad x = 6.46.$$

C $n = 6.46$, the point $n$ is called the *centre of gyration*. When the centre of gyration in a rotating body is known, the accumulated work is readily found by the methods already given. But to find this centre generally, requires the aid of the Integral calculus, which would be foreign to the objects of this work to introduce. The distance of the centre of gyration from the axis, in a few of the most useful cases is as follows :—in a circular wheel of uniform thickness, is equal to the radius of the wheel $\times \sqrt{\frac{1}{2}}$ ; in a rod revolving about its extremity, it is equal to the length of the rod $\sqrt{\frac{1}{3}}$, and when it revolves about its centre, it is equal to the length $\times \sqrt{\frac{1}{12}}$ ; and in a plane ring, like the rim of a fly-wheel, it is equal to the square root of one half of the sum of the squares of the radii forming the ring.

*Ques.* 20. The weight of a fly-wheel is 10 cwt. This weight is collected in a point at a distance of 5 feet, from

the axis. The wheel makes 20 revolutions per minute, the diameter of the axis is 2 inches, and the friction upon it $\frac{1}{7}$ of the whole weight. How many revolutions will the wheel make before it stops ?

Velocity of 10 cwt. or 1120 lbs. per second

$$= \frac{10 \times 3.1416 \times 20}{60} = 10.47 \text{ feet.}$$

Work in the wheel

$$= \frac{\mathbf{10.47^2 \times 1120}}{\mathbf{64\frac{1}{3}}} = \mathbf{1908.43}.$$

Circumference of the axis $= \frac{2}{12} \times 3.1416 = .5236$ feet.

Work destroyed in $x$ revolution

$$= \mathbf{.5236} \times x \times \frac{\mathbf{1120}}{\mathbf{7}} = \mathbf{1908.43}.$$

$\therefore$ $x = 22.78$ revolutions before the wheel comes to rest.

*Ques.* 21. The weight of a fly-wheel is 4 cwt., the distance of the centre of gyration from the axis 4 feet, and the number of revolutions per minute 40 ; find the number of strokes that the fly-wheel will give a forge hammer weighing 200 lbs., with a lift of 2 feet, friction neglected ?

Velocity of the wheel per second $= 16.755$ feet.

Work in the wheel

$$= \frac{\mathbf{16.755^2 \times 448}}{\mathbf{64\frac{1}{3}}} = \mathbf{1954.9}.$$

Let the hammer make $x$ lifts. Work of $x$ lifts of the hammer

$$= \mathbf{200} \times \mathbf{2} \times x = \mathbf{1954.9}.$$

$$\therefore x = \frac{\mathbf{1954.9}}{\mathbf{400}} = \mathbf{4.8}.$$

*Ques.* 22. The diameter of a grindstone is 4 feet, and its weight 300 lbs. The circumference is made to revolve with

the velocity of 5 feet per second. The circumference of the axis is 6 inches, and the friction upon it $\frac{1}{8}$ of the weight. It is required to find the number of revolutions, which the stone will make when left to itself?

Centre of gyration from the axis $= 2\sqrt{\frac{1}{2}}$.

To find the velocity of the centre of gyration, say, as

$$2 : 5 :: 2\sqrt{\tfrac{1}{2}} : 5\sqrt{\tfrac{1}{2}};$$

The square of the velocity of the weight in feet per second

$$= \frac{25}{2}.$$

$$\text{Work in the stone} = \frac{25 \times 300}{2 \times 64\frac{1}{3}} = 58.3;$$

Let $x$ be the number of revolutions, then the work destroyed in $x$ revolutions

$$= \frac{6}{12} \times \frac{300}{8} \times x = 58.3;$$

$$\therefore \quad x = 3.109.$$

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## OF VIS VIVA, OR LIVING FORCE.

THE *vis viva* of a moving body, is a force which is expressed by the product of the quantity of matter by the square of the velocity.

Let the weights of two bodies, A and B, be 4 and 7 lbs. respectively, and their velocities 5 and 9 feet per second; required the accumulated work of the bodies in terms of the *vis viva*?

$$\text{Work in A} = \frac{5^2 \times 4}{2 \times 32\frac{1}{6}}; \quad \text{Work in B} = \frac{9^2 \times 7}{2 \times 32\frac{1}{6}}.$$

$\therefore$ Accumulated work in A and B

$$= \frac{1}{2} \times \frac{5^2 \times 4 + 9^2 \times 7}{32\frac{1}{6}};$$

The latter part of the expression is called the *vis viva* of the bodies A and B.

$$\therefore \text{ Accumulated work} = \tfrac{1}{2} \text{ the } \textit{vis viva}.$$

*Ques.* 23. What must be the weight of another body W, having the velocity of 8 feet per second, so that its accumulated work may be the same as that of the two bodies mentioned above ?

In this case, the work in W

$$= \frac{8^2 \times W}{2 \times 32\tfrac{1}{6}};$$

but this will be equal to the work in the bodies mentioned above ; therefore, by reduction,

$$8^2 \times W = 5^2 \times 4 + 9^2 \times 7$$

$$\therefore \quad W = 10.4 \text{ lbs.}$$

In the above examples, if the bodies A, B, and W, are parts of the same machine, and if the velocity of A be increased, the velocity of B and W will obviously be increased in the same ratio ; hence, it follows, from the expression, that W will not be altered by any change in the velocity of the machine.

---

## OF THE MAXIMUM VELOCITY OF THE PISTON OF AN ENGINE.

THE maximum velocity will obviously take place when the pressure of the steam in the cylinder is equal to the total pressure of the resistances upon the piston ; for, so long as the pressure of the steam is greater than the resistances, the motion of the piston must be accelerated, then the work remaining in the piston, at this point of the stroke, will be equal to the work accumulated in the machine. From this equality the velocity may be found.

*Ques.* 24. The pressure of the steam is 40 lbs., the steam is cut off at 2 feet of the stroke, and the gross load upon the piston 16 lbs. per square inch. At what point in the stroke will the velocity of the piston be the greatest?

Let $x$ = the length of the stroke, then according to Boyle's law, the pressure at this point

$$= \frac{2 \times 40}{x};$$

but the velocity of the piston is greatest when the pressure = 16 lbs. per square inch,

$$\therefore \quad 16 = \frac{2 \times 40}{x}$$

$$\therefore \quad x = 5 \text{ feet.}$$

*Ques.* 25. The length of the stroke in a condensing engine is 10 feet, the pressure of the steam 30 lbs., and the steam is cut off at 2 feet of the stroke. Required the gross load upon each inch, and the point at which the velocity of the piston is greatest?

Dividing the space through which the steam acts expansively into four equal parts, and finding the pressures by Boyle's law, the total work of the steam upon 1 inch of the piston

$$= \frac{2}{3}\left\{30 + 6 + 4\,(15 + 7.5) + 2 \times 10\right\} + 30 \times 2 = 157.333.$$

$\therefore$ Mean pressure of the steam, or gross load

$$= \frac{157.333}{10} = 15.73 \text{ lbs.}$$

Since the mean pressure of the steam is equal to the total resistances which the steam has to overcome throughout the stroke, the maximum velocity will take place when the steam, expanding itself, attains a pressure equal to 15.73 lbs.; hence, by Boyle's law,

$$30 \times 2 = 15.73 \times x,$$

$$15.73 : 30 :: 2 : x;$$

$x$ being the point in the stroke when the velocity is the greatest ;

$$\therefore \quad x = 3.813 \text{ feet.}$$

*Ques.* 26. If the area of the piston in the last example = 4000 square inches, and the weight of the mass moved, calculated to have the same motion as the piston, = 50000 lbs. ; what will be the maximum velocity of the piston ?

Having determined in example 25, the point where the maximum velocity takes place, namely, 3.813 feet, or 4 feet nearly, from the end of the cylinder, the first thing to be done is to find the work of the steam up to this point. Total work upon the whole piston to the point where the maximum velocity takes place

$$= \left[\tfrac{5}{3}\left\{30 + 15 + 4\,(24 + 17.142) + 2 \times 20 + 30 \times 2\right]\right. \times 4000 = 406378 ;$$

the steam acting expansively over 2 feet of the stroke, and dividing this space into four equal parts. Now, the excess of this work over the work which is expended in moving the resistances, will leave us the work which is accumulated in the piston. But the work done upon the resistances

$$= 15.733 \times 4000 = 251733.$$

Work accumulated in the piston

$$= 406378 - 251733 = 154645.$$

Put $v$ = the maximum velocity of the piston in feet per second, then

$$\frac{v^2 \times 50000}{64\tfrac{1}{3}} = 154645.$$

$$\therefore \quad v = 14.1 \text{ feet.}$$

In the same manner the velocity may be found for any part of the stroke. The weight of the mass moved in any engine referred to the piston, may be found as in example 23. The mechanical principles evolved ia the last four examples are worthy of particular attention.

## VELOCITY ACQUIRED BY A BODY DESCENDING AN INCLINED PLANE.

When a body freely descends an inclined plane by the force of gravity alone, the work accumulated in the body, will be simply that which is due to the perpendicular elevation, without any regard to the angle or curve of the plane. Hence, the velocity of the body will be that which it would acquire by falling freely through the perpendicular height.

*Ques.* 27. What velocity will a body acquire in descending an inclined plane, whose perpendicular height is 579 feet?

$$t^2 \times 16\tfrac{1}{12} = 579$$

$$t^2 = 36$$

$$t = 6. \text{ seconds.}$$

Consequently, the velocity acquired at the end

$$= 6 \times 32\tfrac{1}{6} = 193 \text{ feet.}$$

When the friction is taken into account, it will be more convenient to employ the usual expression for accumulated work.

*Ques.* 28. A train of 10 tons descends an incline of 200 feet, having a total rise of 3 feet; what will be the velocity acquired by the train, supposing the friction to be 8 lbs. per ton.

Work in the train

$$= 10 \times 2240 \times 3 - 10 \times 8 \times 200 = 51200.$$

Let $v$ = the velocity,

$$\therefore \quad \frac{v^2 \times 22400}{64\tfrac{1}{3}} = 51200,$$

$$\therefore \quad v = 12.1 \text{ feet.}$$

### TO FIND THE FRICTION UPON AN AXIS BY EXPERIMENT.

*Ques.* 29. The fly-wheel in example 20, was observed to make 28 revolutions before it came to a state of rest; what part of the weight of the wheel is the friction upon the axis?

Let $z$ be put for this proportional part, then the work destroyed by the friction

$$= z \times 1120 \times .5236 \times 28;$$

but this work must be equal to the work accumulated in the wheel, hence,

$$z \times 16420.096 = 1909.43.$$

$$\therefore \quad z = .116.$$

---

### OF THE RESISTANCE OF FLUIDS.

THE resistance of a fluid to the motion of a body is occasioned by the force necessary to displace that fluid. Now the fluid displaced must have the same motion given to it as that of the moving body; hence the work destroyed by the fluid will be equal to the accumulated work in the fluid. Let, for example, the front of the body, presented to the fluid, contain 2 square feet, the weight of a cubic foot of the fluid $62\frac{1}{2}$ lbs., and the velocity of the body 9 feet per second,—then the weight of the fluid displaced every second $= 9 \times 2 \times 62\frac{1}{2}$ lbs., but this weight has a velocity of 9 feet given to it; therefore, the work expended in the displacement

$$= \frac{9^2 \times 9 \times 2 \times 62\frac{1}{2}}{64\frac{1}{3}};$$

Now this work has been destroyed, while the body has moved the space of 9 feet; if $y$ be put for the resistance in lbs. this work will also be represented by

$$y \times 9;$$

$$\therefore\ y \times 9 = \frac{9^2 \times 9 \times 2 \times 62\frac{1}{2}}{64\frac{1}{3}};$$

$$\therefore\ y = \frac{9^2 \times 2 \times 62\frac{1}{2}}{64\frac{1}{3}}.$$

Hence, the resistance increases with the square of the velocity, as well as with the extent of surface presented to the fluid. In extreme velocities this law does not hold strictly true. It appears also from certain recent railroad experiments, that the resistance of the atmosphere to the motion of the train, depends chiefly upon the length of the train, and not so much upon the extent of the frontage of the carriages.

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## OF THE EFFLUX OF FLUIDS.

If an aperture be made in the side of a vessel, kept full of water, then the accumulated work of the fluid, as it is being discharged, will just be equal to the work which gravity would perform upon it, while descending a space equal to the perpendicular depth of the orifice. Hence it follows, that the velocity of the discharge is equal to the velocity which a body would acquire in falling freely through a space equal to the perpendicular depth of the orifice.

*Ques.* 30. The cross sectional area of a pipe is 36 square inches, how many cubic feet will it discharge per minute from a vessel kept constantly full, the depth of the discharge pipe being $16\frac{1}{12}$ feet?

Let $v$ be the velocity of the discharge per second, and $w$ the weight of a fluid particle in lbs., then, the accumulated work in the particle

$$= \frac{v^2 \times w}{64\frac{1}{3}};$$

Work of the particle in descending $16\frac{1}{12}$ feet

$$= 16\frac{1}{12} \times w;$$

$$\therefore \quad \frac{v^2 \times w}{64\frac{1}{3}} = 16\frac{1}{12} \times w;$$

$$\therefore \quad v = \sqrt{16\frac{1}{12} \times 64\frac{1}{2}} = 32\frac{1}{6} \text{ feet.}$$

Discharge per minute, in cubic feet

$$= \frac{36}{144} \times 32\frac{1}{6} \times 60 = 482\frac{1}{2}.$$

*Ques.* 31. If the section of the cistern, in the last example be 8 square feet, and a pressure of 10000 lbs, be applied to a piston fitting the cistern and in contact with the water ; what will be the velocity of the discharge ?

It is evident, that the effect of the pressure of the piston upon the efflux, will be equivalent to an additional column of fluid producing the given pressure on the piston.

Height of additional column

$$= \frac{10000}{8 \times 62.5} = 20 \text{ feet.}$$

Height of the total column of fluid

$$= 16\frac{1}{12} + 20 = 36\frac{1}{12};$$

then proceeding as in the last example, the velocity $v$, will be found

$$= \sqrt{64\frac{1}{3} \times 36\frac{1}{12}} = 48\frac{1}{6} \text{ nearly.}$$

---

## OF THE WORK PERFORMED BY A JET OF STEAM.

When steam issues from a nozzle, its elasticity is nearly the same as that of the surrounding atmosphere ; and the volume of steam at the nozzle will therefore be 1711 times that of the water from which it is raised. The motion of the particles of the steam at the nozzle, is due to the work of the expansion of the steam.

*Ques.* 32. Steam is discharged from a nozzle, whose area is 3 square inches, with the velocity of 600 feet per second ;

how many horse power would a wheel perform, that takes up all the work in the steam?

Cubic feet discharged per second

$$= \frac{3}{144} \times 600 = 12.5.$$

Weight of steam discharged per second

$$= \frac{12.5 \times 62.5}{1711} = .4566 \text{ lbs.}$$

Accumulated work per second

$$= \frac{600^2 \times .4566}{64\frac{1}{3}} = \mathbf{2582}$$

$$\text{Horse power} = \frac{\mathbf{2582 \times 60}}{\mathbf{33000}} = \mathbf{4^{h.}.7}.$$

*Ques.* 33. If 9 cubic feet of water be evaporated per hour, when the area of the nozzle is 1 square inch; how many horse power will the wheel have?

Cubic feet of steam discharged per second

$$= \frac{1711 \times 9}{60 \times 60};$$

Velocity of the steam per second

$$= \frac{1711 \times 9 \times 144}{3600} = 68.4 \times 9;$$

Weight of the steam discharged per second

$$= \frac{9 \times 62.5}{3600} = .017 \times 9.$$

Accumulated work per second

$$= \frac{(\mathbf{68.4 \times 9})^2 \times \mathbf{.017 \times 9}}{\mathbf{64\frac{1}{3}}} = \mathbf{1.23 \times 9^3};$$

From this expression it appears, that the work performed by steam in this manner, increases with the cube of the water evaporated;

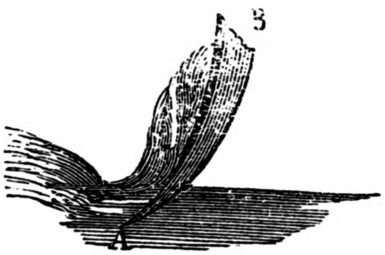
$$\text{Horse power} = \frac{1.23 \times 9^3 \times 60}{33000} = 1^{h.}.6$$

In these calculations the co-efficient of efflux has been neglected.

---

## OF PONCELET'S WATER WHEEL.

In the common under-shot water wheel, the paddles are flat, whereas, in Poncelet's wheel they have a curved shape, A B; so that the direction of the curve at A, where the water meets the paddle, is the same as the direction of the stream.




By this ingenious contrivance, the water rolls up the curved incline A B, without meeting with any sudden obstruction calculated to occasion a loss of work. The channel has a depression at the point where the water falls from the paddles. Let V be the velocity of the steam, and $v$ that of the wheel, then since the point A of the paddle is moving away from the stream, the water will flow upon the paddle with the velocity $V-v$, and will continue to run up the curved line until it has lost its motion, it will then descend, acquiring in its descent the same velocity as that which it had in its ascent, but in a contrary direction. If the wheel were not to move during its return, $V-v$ would be exactly the velocity of discharge from right to left, but the paddle is moving the water from left to right with the velocity $v$, therefore, the absolute velocity of the water upon leaving the paddle will be $V - v - v = V - 2v$.

Now all the work will have been taken out of the water, when its motion upon leaving the paddle is nothing, that is, when $V - 2v = o$,

$$\text{or } V = 2v.$$

In this case the water having lost all its motion, will simply drop from the paddle. and the work done upon the wheel will be equal to the work accumulated in the water

of the stream. Moreover it appears, that the maximum condition is fulfilled when the velocity of the stream is double that of the wheel. However, the distinguished inventor states, that, in practice, the velocity of the water, in order to procure it maximum effect, ought to be $2\frac{1}{2}$ times that of the wheel, and that the *modulus* of the wheel is about $\frac{7}{10}$. This wheel, other things being the same, will perform about twice the work of the common under-shot wheel.

When there is motion in the water after leaving the wheel, the work accumulated in the water must be calculated, and subtracted from the whole work originally in it, in order to obtain the work done upon the wheel.

$$\frac{W \times V^2}{2g} = U + \frac{W(V-2v)^2}{2g};$$

$$\therefore \quad U = \frac{2W}{g}(V-v)v.$$

*Ques.* 34. Suppose 193 cubic feet of water flow upon the paddles of a Poncelet wheel per second, with the velocity of 8 feet., when the wheel moves at the rate of 3 feet per second ; required the horse power of the wheel ?

This is obviously a case of maximum work. Weight of the water flowing per second

$$= 193 \times 62.5 = 12062.5 \text{ lbs.}$$

Work per second

$$= \frac{8^2 \times 12062.5}{64\frac{1}{3}} = 12000;$$

$$\text{Horse power} = \frac{12000 \times 60}{33000} = 21^{\text{h.}}.818.$$

No account is taken of friction and other resistances.

*Ques.* 35. Required the same as in the preceding example, when the quantity of water is 386 cubic feet per second, the velocity of the stream 8 feet, and of the wheel 2 feet ?

Here the velocity with which the water leaves the paddles

$$= 8 - 2 \times 2 = 4 \text{ feet.}$$

The work done upon the wheel per second

$$= \mathbf{18000}.$$

Weight of the water $= 386 \times 62.5$ lbs.

$$\frac{\mathbf{2 \times (386 \times 62.5)}}{\mathbf{32\frac{1}{6}}} \times (\mathbf{8 - 2}) \times \mathbf{2} = \mathbf{18000}''.$$

$$\text{Horse power} = \frac{\mathbf{18000}'' \times \mathbf{60}}{\mathbf{33000}'} = \mathbf{32}^{\text{h.}}\ \mathbf{.72}.$$

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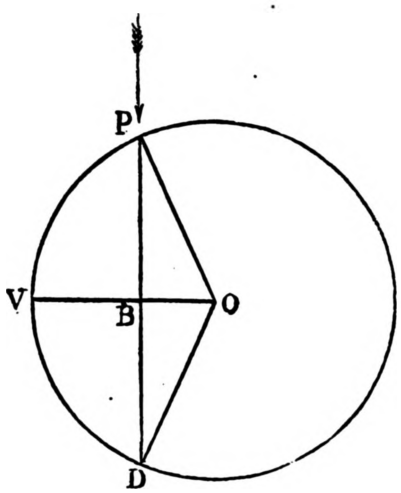
## THE FLY-WHEEL.

*To find the position of the crank corresponding to its maximum and minimum velocity in a single acting engine.*

LET O P, and O D, be the required positions of the crank, and let P be supposed to be the constant pressure of the connecting rod, acting always in a vertical line.




Let Q be the constant resistance, acting at 1 foot from the axis of the fly-wheel, equivalent to the work of the engine. The motion will be accelerated from P to D. This acceleration will commence when the moving pressure is equal to the resisting pressure, and will cease under the same condition. The former will correspond to the position of minimum, the latter to that of the maximum velocity. Hence, at these two points, the moment of P must be equal to the moment of Q, and the point D will be as much below the horizontal line O V, as the point P is above it.

$$\therefore \quad P \times O\,B = Q \times 1 \text{ foot}, = Q \times 1. \quad (y).$$

Again, by the equality of work, and putting $r = V\,O$; and $\pi = 3.1416$, the circumference of a circle when the diameter is unity.

Work of P in one revolution $= 2\,r\,P$;

Work of Q in one revolution $= 2\,\pi\,Q$;

$$\therefore \quad 2\,r\,P = 2\,\pi\,Q; \quad (z).$$

Dividing equation $(y)$ by equation $(z)$,

$$\text{then} \quad \frac{O\,B}{r} = \frac{1}{\pi} = \frac{1}{3.1416} = .3183;$$

this is the cosine of the angle P O V; hence from the tables, angle P O V $= 71°..27'$.

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## TO FIND THE DIMENSIONS OF THE FLY-WHEEL.

Let $d$ and $p$ be the maximum and minimum velocities of the wheel at the distance of 1 foot from the axis; $w$, the weight of the wheel, and $k$ the distance of the centre of gyration from the axis.

Work of P from P to D

$$= P \times P\,D = 2\,r\,P \text{ sin. } 71°..27' = 2\,r\,P \times .948;$$

Work of Q from P to D

$$= \frac{Q \times 2\,\pi \times 142°..54'}{360°} = 2\,r\,P \times .3968, \text{ by sub-}$$

stituting the value of $Q \times 2\,\pi$ given in equation $(z)$.

Now the difference of these will give the work that goes on increasing the speed of the wheel between the points P and D, that is, work going into the wheel between P and D $=$

$$2\,r\,P \times .948 - 2\,r\,P \times .3968 = r\,P \times 1.1022;$$

$$\text{Accumulated work at P} = \frac{p^2 k^2 w}{2g};$$

$$\text{Accumulated work at D} = \frac{d^2 k^2 w}{2g};$$

$$\text{Difference} = \frac{k^2 w}{2g} \times (d^2 - p^2),$$

work gained from P to D.

But this must be equal to the work before found

$$\therefore \quad \frac{k^2 w}{2g}(d^2 - p^2) = r P \times 1.1022; \quad (1).$$

Let $v$ be the mean velocity of the wheel at 1 foot from the axis, and let the velocities $d$ and $p$, at each of the extremes, differ from the mean by the $n$th part; then,

$$d = v + \frac{v}{n} \quad \text{and} \quad p = v - \frac{v}{n};$$

$$\therefore \quad d^2 - p^2 = \frac{4 v^2}{n}; \quad (2).$$

Let $U$ be the work of the engine, and $N$ the number of strokes performed per minute;

$$\text{then } v = 2 \times 3.1416 \times \frac{N}{60} = .10472 \times N; \quad (3).$$

$$\therefore \quad U = 2 r P N \quad \therefore \quad r P = \frac{U}{2 N}; \quad (4).$$

Substituting the values given in equations (2), (3), and (4) in equation (1), and reducing the given quantities, there will be obtained finally

$$\therefore \quad w = \frac{n U}{k^2 N^3} \times 808.2. \quad (5).$$

After the same manner may be derived an equation, expressing the relation of the elements for the double acting engine.

*Ques.* 36. In a single acting engine of 30 horse power, the number of strokes performed by the piston = 20 per minute, the distance of the centre of gyration of the fly-wheel from the axis = 10 feet; it is required to find the weight of the wheel, so that the velocity of each revolution may not vary by more than $\frac{1}{5}$ from the mean?

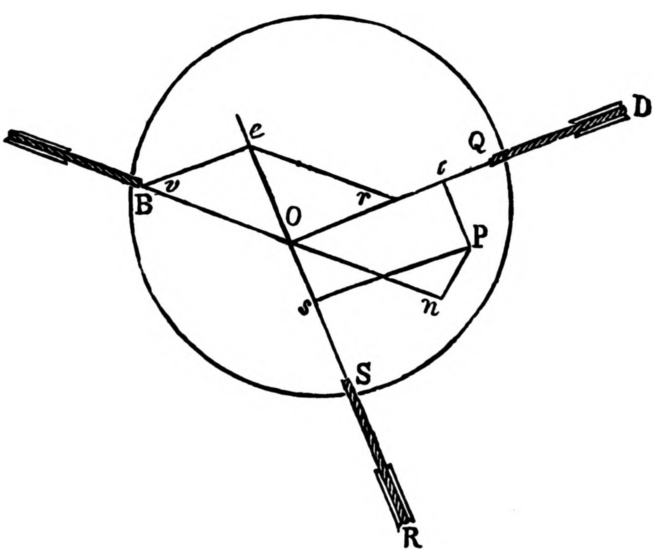
$$n = 5\ ;\ \boldsymbol{U = 30 \times 33000}\ ;\ k = 10\ ;\ N = 20.$$

$$W = \frac{5 \times 990000}{10^2 \times 20^3} \times 808.2 = 50007.4 \text{ lbs.}$$

## CHAPTER VII.

### OF THE EQUILIBRIUM OF FORCES AND PRESSURES.

LET the board A W R, be placed upon three balls rolling freely upon a horizontal table. Take any three points, B, Q, S, in the surface of the board, and let the cords A B,




D Q, R S, be attached to the points. These cords pass over pulleys, and have different weights hanging at their extremities. Under this arrangement the board, acted

upon by three forces, will roll upon the table until it comes to a position where the three forces destroy each other. When this position has been attained, the following important relations will be observed, between the direction and magnitude of the forces.

1. The directions of the forces will intersect, or meet, in the same point O.

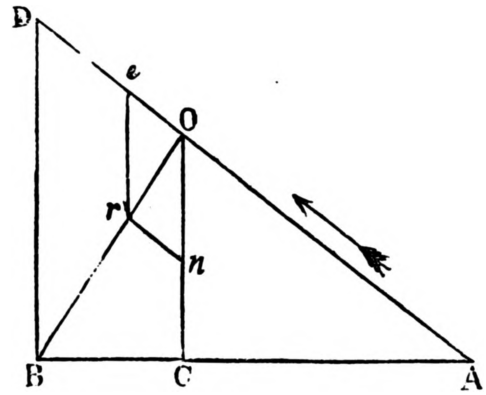
2. From a scale of equal parts, take off O *r*, equal to the units of pounds in the weight or force, drawing the cord Q D, and in like manner from the same scale of equal parts take off O *v*, equal to the units of pounds in the weight, or force, drawing the cord A B, then if upon the two lines O *r*, O *v*, a parallelogram O *r e v* be constructed, the diagonal O *e* will be the direction of the third force acting by the cord R S, and the units of length in the diagonal O *e*, will give the units of pounds in the weight, or force, acting by the cord R S. The forces represented by the sides O *r*, and O *v*, are called the component forces, and the force just equivalent to these two and represented by O *e*, is termed the resultant.

3. Instead of three forces being applied to the board, let there by any number. Let P be any point taken in the board, and from this point let fall the perpendiculars P *n*, P *s*, P *t*, &c., on the directions of the forces, or if necessary, on the directions of the forces produced; then the units of length in any of these perpendiculars, multiplied by the units of pounds in the corresponding force, will be the moment of that force, tending to turn the board upon the point P; and then the principle of the equality of moments will be obtained.

$$P\,n \times O\,B + P\,t \times O\,r = P\,s \times O\,e.$$

### EXAMPLES.

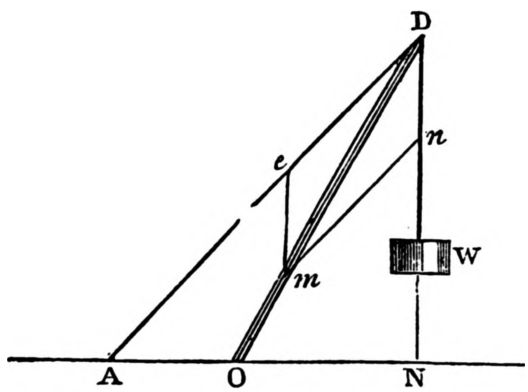
*Ques.* 1. Let A B, be a platform with a load C upon it, supported by a chain A D; it is required to determine by construction the tension of the chain, and also the amount and direction of the pressure upon the hinge at B, the weight of the platform being neglected?




CONSTRUCTION.

THROUGH the point C, draw the vertical line C O, intersecting the direction of the chain in the point O, join B and O, and from a scale of equal parts set off O *n* equal to the units in the weight placed at C ; draw *r n* parallel to A D, and *r e* parallel to C O, then O *n r e* will be the parallelogram of pressures. Because, O is the intersection of two of the forces or pressures ; therefore, the direction of the third force must pass through O ; but this third force must also pass through B. Hence it follows, that O B is the direction of the pressure upon the hinge, and in like manner O *e*, will give the units of pressure tending to break the chain.

*Ques.* 2. A pole, O D, supported by a cord A D, carries a weight W ; it is required to find the tension of the cord, and the pressure on the pole ?



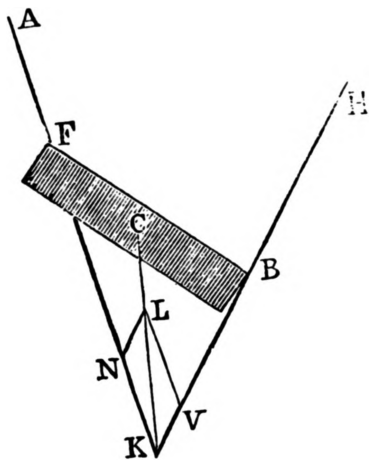

CONSTRUCTION.

On the line D N, mark off D *n* equal to the units of

weight in W, draw $n\,m$ parallel to D A, intersecting D O in $m$, and from $m$ draw $m\,e$, parallel to D $n$; then D $n\,m\,e$ will be the parallelogram of pressures; therefore, the units in D $e$ will give the tension of the cord, and the units in D $m$ will give the pressure upon the pole.

*Ques.* 3. A beam, F B, is supported by a cord F A; it is required to determine by construction, the direction and tension of the cord B H, so that the beam may not change its position?

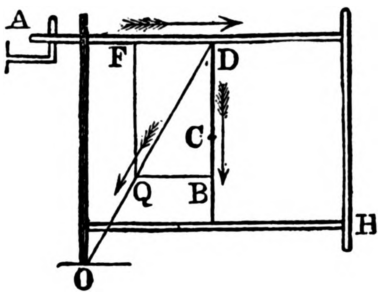
CONSTRUCTION.




THROUGH the center of gravity, C, draw the vertical line C K, produce A F, until it intersect this vertical line in the point K, join K and B, then K B produced, will give the *direction* of the cord. Take K L, equal to the units in the weight of the beam, and construct the parallelogram of pressures, K N L V, then the units in K V, will give the tension of the cord H B.

*Ques.* 4. A gate, A H, is supported by a pin turning in a socket at O, and prevented from falling in the direction A D by a hook and loop at A; it is required to determine the amount and direction of the pressure upon the pin O, and the force tending to draw the hook A from the wall?

CONSTRUCTION.




THROUGH the middle, or center of gravity of the gate draw the vertical line D C, join the points D and O, then D O will be the direction of the pressure upon the pin. Take D B equal to the units in the weight of the gate, and construct the parallelogram of pressures F D B Q, then the

units in D Q will be the pressure upon the pin, and the units in D F will be the force tending to draw the hook.

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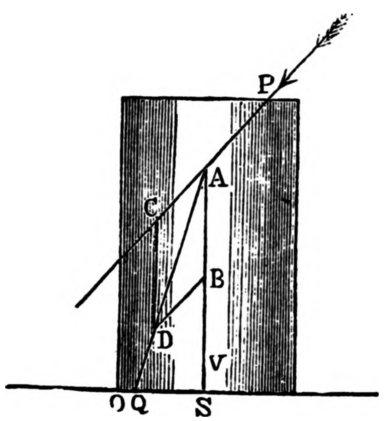
## TO FIND WHETHER A PILLAR WILL STAND OR FALL WHEN ACTED UPON BY A GIVEN PRESSURE.

*Ques.* 5. LET P be the direction and amount of the pressure, tending to turn the pillar upon the edge O as a centre; and A V a vertical line passing through the centre of gravity of the pillar, intersecting the line P, produced in the point A; from a scale of equal parts, take A C equal to the units in the pressure P, and from the same scale take A B, equal to the units of weight in the pillar; construct the parallelogram of forces A B D C, then A D will be the amount and direction of the single force tending to over-




turn the pillar. If A D produced, intersect the base within the edge O, the pillar will stand; and on the contrary, if the point of intersection Q, fall without the base, the pillar will fall; but if it intersect at the edge O, then the pillar will just be upon the point of overturning; the point Q is called *the point of resistance.*

A structure will be more or less stable, according as the *point of resistance*, Q, is more or less distant from the edge O. Hence, the *modulus of stability*, may be defined to be, the ratio that O Q bears to O S. Thus, if Q were in the middle between O and S, the modulus would be $= \frac{1}{2}$; if Q

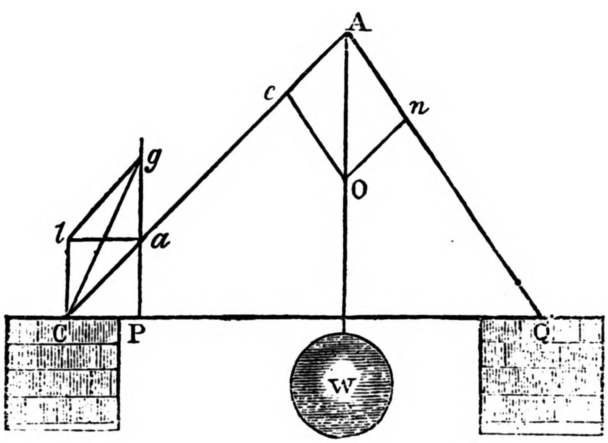
were at S, the modulus would be 1, or the most stable possible, under the given conditions; and if Q were at O, the modulus would be zero, that is, the structure would be on the point of overturning. *Marshal Vauban* considered, that the modulus of a good structure ought to be about $\frac{1}{2}$.

## PRESSURE OF ROOFS.

*Ques.* 6. LET C A, and Q A, be the rafters of a roof resting upon the side walls C and Q. It is required to determine the thrust on the points C and Q, the roof being without a tie beam?

### CONSTRUCTION.

Let it be supposed, that the weight of the roof is equally distributed over the surface, and that a foot of length of this roof acts upon a foot of length of the side walls.




Find the weight of each side of the roof one foot long, and let a weight W, be suspended from A equal to half the sum of these weights, then one half of the whole weight upon the rafter, C A, will act perpendicularly upon the wall at C, the other half acting in the weight W; and in like manner one half of the whole weight upon the rafter, Q A, will act perpendicularly upon the wall at Q, the other half acting in the weight W; now take O A, equal to the units of weight in W, and construct the parallelogram of pressures A *e*, O *n*, then A *e* will give the thrust upon the rafter A C, and A *n* the thrust upon the rafter A Q.

*Ques.* 7. *To find the tension of the tie beam Q C, or of a cord connecting the feet of the rafters ?*

CONSTRUCTION.

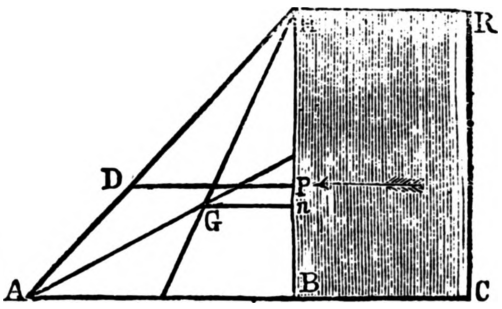
Take C $a$ = A $e$, the thurst upon the rafter A C, draw $a$ P perpendicular to C Q, and construct the parallelogram at C P, then, the pressure C $a$ is equivalent to the two pressures, C $t$, and C P ; now the latter pressure gives the tension of the cord which would be produced by the thurst of the rafter C A ; but the rafter A Q, will produce an equal and opposite tension, therefore, the units in 2 × C P, will give the units of tension of the cord C Q.

*Ques.* 8. *To find the amonnt and direction of the pressure of the roof, tending to overturn the side walls ?*

It has been shown, that besides C A, the thrust upon the rafter, we have a vertical pressure upon the wall, arising from one half the weight borne by the rafter. Take C $t$, equal to the units in this vertical pressure, and construct the parallelogram C $a$ $q$ $t$, then the diagonal C $q$, will give the amount and direction of the pressure of the roof tending to overturn the wall C.

*Ques.* 9. *Pressure of fluids on Embankments.*

Let H B, be the side of a vessel filled with water, then the pressure of the fluid upon the point P, which may be




any point in the side, will be due to the perpendicular depth H P. That is, in other language, the pressure is in proportion to the depth. The reason of this is, since gravity

acts upon all the particles of the fluid, each particle presses on that which is next below it, and then, from the peculiar property of a fluid, this pressure is transmitted equally in every direction.

In the base B C produced, take B A, equal to B H, the perpendicular depth of the fluid, then the pressure upon the point B will be due to the pressure of a column of fluid whose height is B A. Join A and H, and from any point, P, draw P D perpendicular to B H, and because P D = P H, the pressure upon the point P, will be due to a column of fluid whose height is P D, and so on to any other point in the side of the vessel. Let us now suppose, that the depth of the water, H B, is 8 feet, and the length of the side 6 feet, then the whole pressure upon the side will be equivalent to the pressure, or weight, of a mass of fluid of the form of the wedge, A B H; area of the triangle, A B H

$$= 8 \times \frac{8}{2} = 32.$$

Contents of the wedge $= 6 \times 32 = 192$ cubic feet.

Pressure $= 192 \times 62.5 = 12000$ lbs.

It may be observed, that the pounds pressure upon the side, or proposed surface of a vessel, is equal to the weight of a column of fluid, whose base is the a rea of the surface, and perpendicular height, the depth of the centre of gravity, or middle point of that surface.

$\frac{8}{2}$ depth of centre of gravity;

$8 \times 6 =$ area of surface.

$\therefore \quad 8 \times 6 \times \frac{8}{2} \times 62.5 = 12000$ lbs. as before.

*Ques.* 10. Find the pressure on a flood-gate whose breadth is 8 feet, and depth 6 feet?

$$\text{Pressure} = 6 \times 8 \times \frac{6}{2} \times 62.5 = 9000 \text{ lbs.}$$

*Ques.* 11. The depth of water pressing against an em-

bankment is 9 feet ; required the pressure upon each foot of length ?

$$1 \times 9 \times \frac{9}{2} \times 62\tfrac{1}{2} = 2531\tfrac{1}{4} \text{ lbs.}$$

*Ques.* 12. Compare the pressure upon the sides of a cubical vessel filled with water, with the pressure upon the bottom, allowing that the side of the base = a foot ?

$$\text{Pressure on the base} = a \times a \times a \times 62.5 ;$$

$$\text{Pressure on the sides} = a \times a \times \frac{a}{2} \times 62.5 \times 4 ;$$

$$\therefore \frac{\text{Pressure on the sides}}{\text{Pressure on the base}} = 2,$$ that is, the pressure on the sides is twice that on the base.

*Ques.* 13. Required the pressure on the staves of a cylindrical barrel filled with water, the diameter of the base being 3 feet, and the perpendicular height 4 feet ?

$$\text{Pressure} = 3 \times 3.1416 \times 4 \times \frac{4}{2} = 4712.4 \text{ lbs.}$$

It will be readily seen, that there must be a certain point in the side of the vessel when filled with water, where a single pressure will exactly counterbalance the pressure of the water against the whole side. This point is called the *centre of pressure.* It must obviously lie in the line *n* G, passing through the centre of gravity G, of the wedge of pressures A H B.

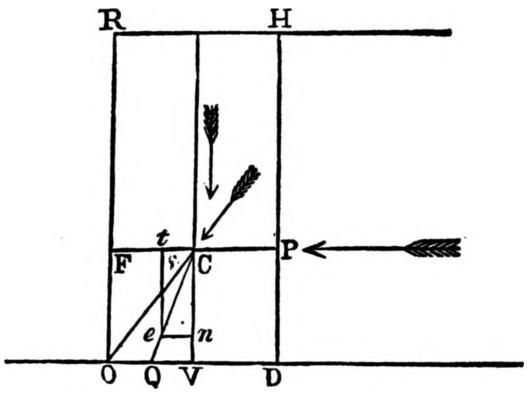
But, B *n* = $\frac{1}{3}$ B H, that is, the centre of pressure *n*, lies at one third of B H, from the bottom of the vessel.

If the staves of a barrel be kept together by a single hoop, and if the barrel be filled with water, then the hoop must be placed at one third from the bottom.

*Ques.* 14. An embankment R D, sustains the pressure of water, whose centre of pressure is at P ; it is required to determine the conditions of equilibrium, &c., supposing the embankment to turn over upon O, as a centre ?

Let V C, be the vertical line passing through the centre

of gravity of the embankment; P the centre of pressure of the water. Draw P F perpendicular to R O, then the product of the pressure of the water by O F, will give the moment of the water, tending to turn the embankment over




upon O as a centre; and in like manner, the product of the weight of the embankment by O V, will give the moment of the embankment, tending to turn itself in a direction opposed to the pressure of the water. When these moments are equal, the embankment is upon the point of overturning. If the moment of the water be greater than that of the embankment, the structure will fall, and *vice versâ*. In the following examples, the length of the embankment is taken at one foot, because, if it stand for one foot, it will stand for any other length.

*Ques.* 15. Let O R = 9 feet, O D = 3 feet, and the weight of a cubic foot of the material 150 lbs.; will the embankment stand or fall when the water is up to the top?

Surface upon which the water presses = $9 \times 1$.

Pressure of the water = $9 \times 1 \times \frac{9}{2} \times 62.5 = 2531\frac{1}{4}$ lbs.

Distance of the centre of pressure from the bottom

$$= \text{O F} = \frac{9}{3} = 3 \text{ feet};$$

Moment of the water

$$= 2531\frac{1}{4} \times 3 = 7593.75.$$

Weight of the embankment $= 3 \times 1 \times 9 \times 150 = 4050$ lbs.

$$OV = \frac{3}{2} = 1.5 \text{ feet.}$$

Moment of the embankment

$$= 4050 \times \frac{3}{2} = 6075.$$

So that the moment of the water is greater than that of the embankment ; hence the structure will fall.

It will be found when the height is 12 feet, the thickness 5 feet, and the weight of a cubic foot of the material 120 lbs., the embankment will be just upon the point of overturning

*Ques.* 16. What must be the height of the water in the last example, so that the embankment may be upon the point of overturning ?

Let $x$ be the height, then the moment of the water

$$= x \times 1 \times \frac{x}{2} \times 62.5 \times \frac{x}{6} = \frac{x^3}{6} \times 62.5 ;$$

$$\therefore \frac{x^3}{6} \times 62.5 = 6075 ;$$

$$\therefore x = 8.3 \text{ feet.}$$

*Ques.* 17. Required the base, or thickness of a rectangular embankment, when the height is 15 feet, the weight of a cubic foot of the material 140 lbs., and the water stands at the brim, so that the structure may be upon the point of overturning ?

Pressure of the water

$$= 1 \times 15 \times \frac{15}{2} \times 62.5 = 7031\tfrac{1}{4} \text{ lbs.}$$

$$\text{Moment} = \frac{15}{3} \times 7031\tfrac{1}{4} = 35156.25.$$

Let $x$ be the required thickness, then the moment of the embankment

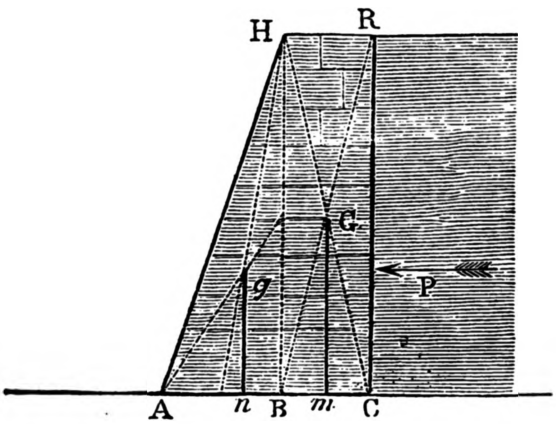
$$= 15 \times x \times 1 \times 140 \times \frac{x}{2} = x^2 \times 1050 ;$$

$$\therefore\ x^2 \times 1050 = 35156.25$$

$$\therefore\ x = 5.79 \text{ feet.}$$

*Ques.* 18. Let the embankment have the form of a trapezoid, A H R C, where A B = 3 feet; B C = 2 feet; R C = 9 feet, and the weight of a cubic foot of the material = 100 lbs.; will the embankment stand or fall, when the water is at the brim?

Let the embankment be divided into two parts, namely, the rectangular R B, and the triangular part A B H. Now, the vertical line passing through the centre of gravity of the triangle, will cut




the base at *n*, 2 feet from the centre of motion A; and a line through the centre of gravity of the parallelogram H C, will cut A C at the distance of 4 feet from A.

$$\text{Weight of A B H} = \frac{3 \times 9}{2} \times 1 \times 100 = 1350 \text{ lbs.}$$

Moment of A B H = $1350 \times 2 = 2700$;

Weight of B H R C = $2 \times 9 \times 1 \times 100 = 1800$ lbs.

Moment of B H R C = $1800 \times 4 = 7200$;

Consequently, the moment of the whole embankment

$$= 2700 + 7200 = 9900.$$

The moment of the water

$$= 9 \times 1 \times \frac{9}{2} \times 62\tfrac{1}{2} \times \frac{9}{3} = 7593\tfrac{3}{4};$$

hence it follows that the embankment will stand.

*Ques.* 19. Required the modulus of stability of the structure, O R H D, (fig. example 14,) when O D = 5 feet ; D H = 9 feet ; the weight of a cubic foot of the material in O R H D = 150 lbs. ; the water is up to the top ?

The weight of the structure

$$= 5 \times 1 \times 9 \times 150 = 6750 \text{ lbs.}$$

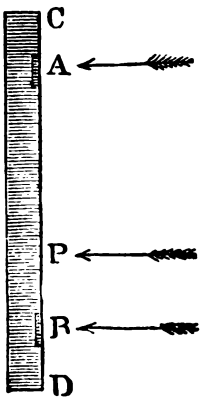
The pressure of the water = 2531¼ lbs.

Draw the rectangle R D, take D P = 3, draw P F parallel to O D, intersecting P F in the point C ; from any diagonal scale mark off C $n$ = 6750 and C $t$ = 2531 ; construct the parallelogram $n\,t$, — then the diagonal C $e$ produced, will intersect the base in Q, the ratio of O Q to O V will be found to be about 13 to 25.

*Ques.* 20. The breadth of a flood-gate is 10 feet, and the depth 6 feet, the hinges are placed at 1 foot from the respective extremities of the gate. It is required to find the pressure upon the lower hinge ?

In this example, the pressure of the water on each half of the gate

$$= 6 \times 5 \times \frac{6}{2} \times 62.5 = 5625 \text{ lbs.}$$




Let C D be the height of the gate, A and B the hinges, and P the centre of pressure of the water ; then D P = $\frac{6}{3}$ = 2 ; A P = 5 — 2 = 3 ; and B A = 6 — 2 = 4.

Now, since the pressure of the water at P is supported by the hinges A and B, then by the principal of the lever supposing A the fulcrum.

Pressure on B × 4 = pressure on P × 3.

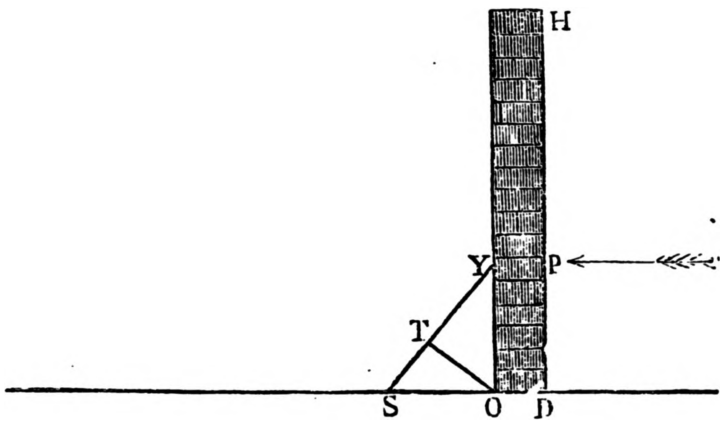
Call $x$ the pressure on B.

$$\therefore \quad 4x = 5625 \times 3$$

$$\therefore \quad x = 4218 \text{ lbs.}$$

*Ques.* 21. Let H D = 12 feet, O D = 1.5 feet, weight of a cubic foot of the material = 130 lbs., S Y = 5 feet, a

shore, or stay supporting the embankment ; S O = 3 feet ; required the thrust upon the stay, when the embankment is upon the point of overturning on O, the water being at the top ?




When the embankment is supported by a shore, or stay, S Y ; and it is required to find the thrust upon this stay when the embankment is upon the point of overturning on the edge O.

Weight of the embankment

$$= 1.5 \times 1 \times 12 \times 130 = 2340 \text{ lbs.}$$

Moment of the embankment

$$= 2340 \times \frac{1.5}{2} = 1755.$$

Let O T be perpendicular upon the stay, then O T = 2.4 feet. Put $z$ = the thrust on the stay, then the moment of the thrust on the stay

$$= z \times 2.4 ;$$

The moment of the pressures of the water

$$= 12 \times 1 \times \frac{12}{2} 62.5 \times \frac{12}{3} = 18000.$$

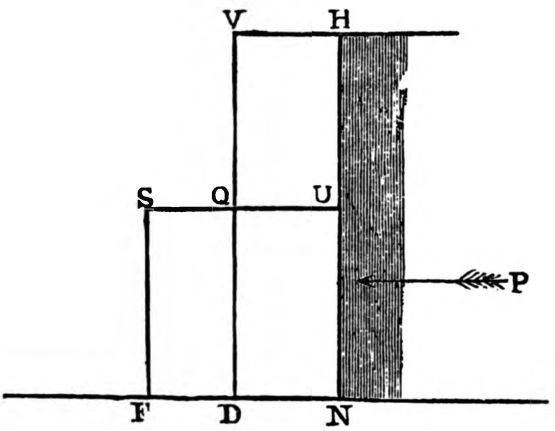
$$\therefore \quad z \times 2.4 + 1755 = 18000.$$

$$\therefore \quad z = 6768 \text{ lbs.}$$

*To determine the form of maximum, or equable strength of an embankment.*

THE embankment, V H N F, will have a maximum strength, if the part V U, be upon the point of overturning upon the edge Q, at the same time that the portion V H N F is upon the point of overturning upon F, by the pressure of the fluid.

*Ques.* 22. The height of the embankment, H N = 6 feet, V Q = 3 feet, and the weight of a cubic foot of the material = 120 lbs.; it is required to find the base F N, so that the embankment may be upon the point of overturning on F, by the pressure of the fluid, at the same time that the part V U is upon the point of overturning on Q, when the water stands at the brim?




The base Q U, must be first found, so that U V may be upon the point of overturning on Q, by the pressure of the water on H U.

Put $x$ = Q U, then since the moments of the water and of the wall are

$$= x \times 3 \times 1 \times 120 \times \frac{x}{2} = 1 \times 3 \times \frac{3}{2} \times 62.5 \times \frac{3}{3};$$

$$\therefore \quad 180\, x^2 = 281.25$$

$$\therefore \quad x = 1.2514 \text{ feet.}$$

To find F N, put F D = $y$, then, the moment of F Q

$$= y \times 1 \times 3 \times 120 \times \frac{y}{2};$$

Moment of V D N H

$$= 1.25 \times 1 \times 6 \times 120 \times \left(y \times \frac{1.25}{2}\right);$$

The moment of the water

$$= 6 \times 1 \times \frac{6}{2} \times 62.5 \times \frac{6}{3} = 2250.$$

Now the sum of the two former moments must be equal to the moment of the water, hence by reduction

$$180\,y^2 + 900\,y + 562.5 = 2250.$$

Solving this equation, it will be found that $y = 1.45$, and F N

$$= 1.45 + 1.25 = 2.7 \text{ feet.}$$

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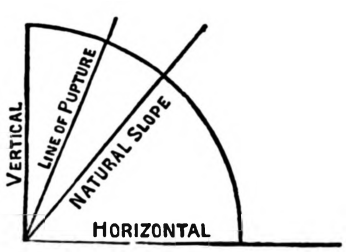
## REVETMENT WALLS.

A WALL sustaining the pressure of earth, or any loose material, is called a revetment wall. The pressure of earth upon a wall is similar to the pressure of water, with only this difference, that the weight of the material must be reduced by a certain ratio depending upon the angle of its natural slope. The thrust of earth upon a wall is occasioned by a certain portion, of a wedge shape, tending to break away from the general mass. Coulomb showed, that the angle which this line of rupture makes with the vertical, is one half the angle which the line of natural slope makes with the vertical. Now when the earth is level with the top, the pressure of earth may always be found by regarding it as a fluid, having the weight a cubic foot, equal to the weight of a cubic foot of the earth, multiplied by the square of the tangent of half the angle of the natural slope from the vertical. In earth of mean quality, the angle of natural slope is about 45°, then the square of the tangent of half this angle will be = .1716, this number, therefore, multiplied by the weight of a cubic



of the material, gives the weight, in this case, of a cubic foot of the equivalent fluid pressing against the wall.

*Ques.* 23. A revetment wall 40 feet high, and 10 feet thick, sustains the pressure of earth of mean quality, having the weight of a cubic foot equal to 100 lbs. ; it is required to determine whether or not the wall will stand, taking the weight of a cubic foot to be 120 lbs. ?

Weight of a cubic foot of equivalent fluid, acting at the back like water

$$= 100 \times .1716 = 17.16 \text{ lbs.}$$

Pressure of the earth

$$= 40 \times 1 \times \frac{40}{2} \times 17.16 = 13728 \text{ lbs.};$$

Moment of the earth

$$= 13728 \times \frac{40}{3} = 183040;$$

Moment of the wall

$$= 1 \times 40 \times 10 \times 120 \times \frac{10}{2} = 240000;$$

Consequently the wall will stand

*Ques.* 24. What is the thickness of the wall, in the last example, so that it may be upon the point of overturning ?

Let $x$ = the required thickness, then the moment of the wall

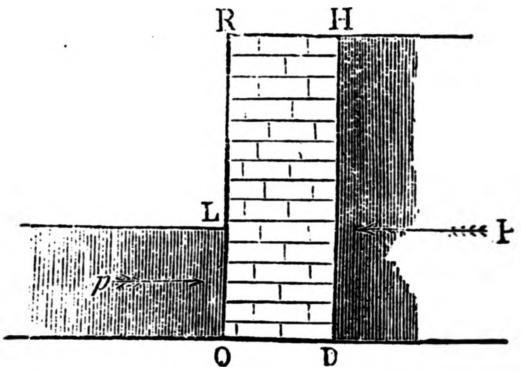
$$= 40 \times 1 \times x \times 120 \times \frac{x}{2} = 2400\, x^2;$$

when the wall is upon the point of overturning, the moment of the wall, must be equal to the moment of the pressure of earth.

$$\therefore \quad 2400\, x^2 = 183040$$

$$\therefore \quad x = 8.7 \text{ feet.}$$

*Ques.* 25. Let O R = 9 feet, O D = 3 feet, the weight of a cubic foot of the material in R D = 150 lbs., the water is to the top H ; besides, the embankment is supported by




earth L O, of mean quality, L O = 3 feet, and level at the top; will the embankment stand, when the weight of a cubic foot of the earth = 120 lbs. ?

It has also been found by experiment, that when a wall is supported by the pressure of earth, the weight of the equivalent fluid is about 6 times the weight of the earth.

Consquently, the earth L O, may be considered a fluid, the weight of a cubic foot of which $= 6 \times 120 = 720$ lbs.

$\therefore$ The moment of this earth

$$= 3 \times 1 \times \frac{3}{2} \times 720 \times \frac{3}{3} = 3240 ;$$

Moment of the wall O H

$$= 3 \times 1 \times 9 \times 150 \times \frac{3}{2} = 6075 ;$$

Moment of the water

$$= 9 \times 1 \times \frac{9}{2} \times 62.5 \times \frac{9}{3} = 7593.75 ;$$

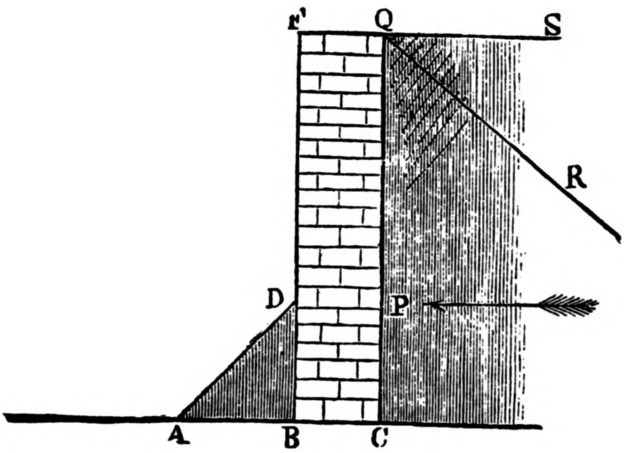
$\therefore$ 3240 + 6075 is greater than 7593.75,

and hence the wall R H D O will stand.

In these examples, the earth is supposed to be level at the top, but when the earth has its natural slope, a similar method of calculation will apply. When the wall resists the pressure of the earth, take $\frac{1}{8}$ of the weight of the earth, for the weight of the equivalent fluid ; and on the contrary,

when the earth resists the pressure of the wall. take ½ the weight of the earth for the weight of the equivalent fluid.

*Ques.* 26. Required the same as in example 23, when the earth has its natural slope Q R, and the thickness of the wall = 8 feet = B C ?




On this supposition, the weight of a cubic foot of the equivalent fluid

$$= 100 \times \tfrac{1}{8} = 12\tfrac{1}{2} \text{ lbs.}$$

The moment of the earth C Q R

$$= 40 \times 1 \times \frac{40}{2} \times 12\tfrac{1}{2} \times \frac{40}{3} = 133333 ;$$

The moment of the wall

$$= 40 \times 8 \times 1 \times 120 \times \frac{8}{2} \quad 153600 ;$$

Consequently, the wall will stand.

*Ques.* 27. A revetment wall, F Q B C, is 30 feet high = B F ; B C = 6 feet. On one side, C Q, earth of mean quality is sustained level with the top, and on the other side, F B, the earth has a natural slope A D, and rises to the height B D = 5 feet ; will the wall stand or fall, supposing that the weight of a cubic foot of the earth to be 120 lbs., and that of the wall 130 lbs. ?

In this case, the moment of the earth, which rises to the level of the wall, is opposed by the sum of the moments of the wall, and the earth on the other side having a natural slope.

Weight of the wall

$$= 30 \times 1 \times 6 \times 130 = 23400 \text{ lbs.}$$

Moment of the wall

$$= 23400 \times \frac{6}{2} = 70200 ;$$

Weight of a cubic foot of the equivalent fluid, with respect to the sloping earth

$$= 120 \times \tfrac{1}{2} = 60 \text{ lbs.}$$

Pressure of this earth

$$= 5 \times 1 \times 60 \times \frac{5}{2} = 750 \text{ lbs.}$$

$$\text{Moment} = 750 \times \frac{5}{3} = 1250 ;$$

Total moments sustaining the wall

$$= 70200 + 1250 = 71450$$

Moment of the level earth

$$= 30 \times 1 \times \frac{30}{2} \times 120 \times .1716 \times \frac{30}{3} = 92664 ;$$

Since the latter moment is greater than the sum of two the former, it may be concluded that the structure will fall.

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## OF FLOATING BODIES, AND SPECIFIC GRAVITY.

When a body floats, it is sustained by the upward pressure of the fluid ; and as there is an equilibrium of pressures, the upward pressure must be equal to the gravity of the floating body. But this upward pressure would just support a volume of fluid equal to that which is displaced, therefore, the weight of the floating body must be equal to the weight of the fluid displaced.

Hence it follows, that if a heavy body be weighed in water, the weight which is lost will be equal to the weight of water having the same bulk or volume as the body. In this way we are enabled to find the specific gravity of a body, or its weight, as compared with the weight of an equal bulk of water. As the weight of a cubic foot of water is just 1000 ounces, it is customary to consider the specific gravity of a body as the weight of a cubic foot of it. The following table contains the weight of a cubic foot of various kinds of material in construction.

| NAME OF THE MATERIAL. | SPECIFIC GRAVITY, OR THE WEIGHT OF A CUBIC FOOT IN OUNCES. | NAME OF THE MATERIAL | SPECIFIC GRAVITY, OR THE WEIGHT OF A CUBIC FOOT IN OUNCES. |
|---|---|---|---|
| Ash,..... | 760 | Brass, cast..... | 8399 |
| Beech,..... | 690 | Iron, wrought..... | 7700 |
| Deal,..... | 698 | Iron, cast..... | 7066 |
| Deal, Memel..... | 590 | Lead, cast..... | 11332 |
| Fir, American..... | 553 | Oak, English..... | 934 |
| Fir, Riga..... | 753 | Teak..... | 657 |

*Ques.* 28. A barge, (supposed for the sake of simplicity, to be of a rectangular shape), is 10 feet long, 5 feet broad, and 4 feet deep, outside measure, the thickness of the planking is 2 inches, and the weight of a cubic foot of the timber is 50 lbs., to what depth will the barge sink when loaded with 4 tons?

Content of the exterior solid

$$= 10 \times 5 \times 4 = 200 \text{ cubic feet.}$$

Content of the interior solid

$$= 9\tfrac{2}{3} \times 4\tfrac{2}{3} \times 3\tfrac{5}{6} = 172.92.$$

200

172.92

Timber in the barge, 27.08 cubic feet.

Weight of timber in the barge

$$= 27.08 \times 50 = 1353 \text{ lbs.}$$

Let $x$ be the depth of the water displaced, then the weight of the displaced water

$$= x \times 10 \times 5 \times 62.5 = 3125\,x;$$

$$\therefore \quad 3125\,x = 1353 + 4 \times 2240$$

$$\therefore \quad x = 3.3 \text{ feet.}$$

*Ques.* 29. How many tons will just sink the barge in the last example?

Let $y$ = the number of tons;

$$200 \times 62.5 = 12500;$$

$$\therefore \quad 12500 = 1353 + y \times 2240$$

$$\therefore \quad y = 4.976 \text{ tons.}$$

*Ques.* 30. What weight will sink the barge in example 28, to the depth of 3 feet?

$$10 \times 5 \times 3 \times 62.5 = 9375.$$

Let the required load $= z$, then

$$9375 = 1353 + z \times 2240$$

$$\therefore \quad z = 3.58 \text{ tons.}$$

*Ques.* 31. How far will the barge sink when empty?

Put $v$ = the required depth, then

$$10 \times 5 \times v \times 62.5 = 3125\,v;$$

$$\therefore \quad 3125\,v = 1353$$

$$v = .43296 \text{ feet.}$$

*Ques.* 32. A body weighs 49 grains in air, and 42 grains in water; what is the weight of a cubic foot of the substance?

The loss of weight

$$= 49 - 42 = 7.$$

This loss is the weight of water having the same bulk as the body.

Consequently, the number of times the body is heavier than water

$$= \frac{49}{7} = 7 \text{ times};$$

But the weight of a cubic foot of water = 1000 ounces, therefore, the weight of one cubic foot of the body = 7 times 1000 ounces = 7000 ounces, and this number is called the specific gravity of the body.

*Ques.* 33. A cubic foot of timber sinks 9 inches in water; required the specific gravity?

In this case, the weight of the displaced fluid will be the weight of a cubic foot of the timber.

Therefore, the specific gravity

$$= 1 \times 1 \times \frac{9}{12} \times 1000 = 750.$$

*Ques.* 34. A Solid, whose weight is 60 grains, weighs 40 grains in water, and 30 grains in sulphuric acid; required the specific gravity of the acid?

Weight lost in water = 60 — 40 = 20 grains.

Weight lost in acid = 60 — 30 = 30 grains.

Therefore, the number of times the acid is heavier than water

$$= \frac{30}{20} = 1\tfrac{1}{2};$$

Hence, the specific gravity of the acid

$$= 1\tfrac{1}{2} \times 1000 = 1500.$$

*Ques.* 35. A piece of metal weighing 36 lbs. in air, and 32 lbs. in water, is attached to a piece of wood whose weight is 30 lbs., and then the compound mass is found to weigh 12 lbs. in water; required the specific gravity of the wood?

Weight of water, equal in bulk to the metal

$$= 36 - 32 = 4 \text{ lbs.}$$

Weight of water, equal in bulk to the compound

$$= 36 + 30 - 12 = 54 \text{ lbs.}$$

Weight of water equal in bulk to the wood

$$= 54 - 4 = 50 \text{ lbs.}$$

But the weight of the wood is 30 lbs., that is bulk for bulk, the wood will be three-fifths of the weight of the water. Consequently, the specific gravity of the wood

$$= \frac{3}{5} \times 1000 = 600.$$

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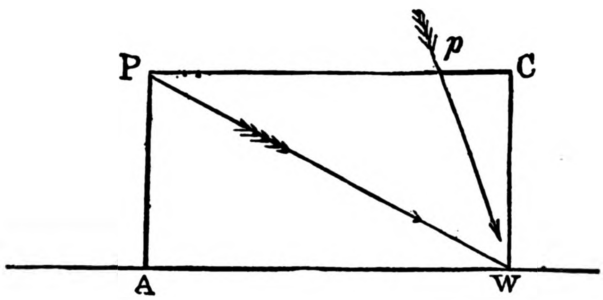
### OF THE LIMITING ANGLE OF RESISTANCE.

Let W be a material particle acted upon by a pressure P, in the direction of and equal in magnitude to P W.

Complete the rectangle A C, then will C W be the pres-




sure upon the plane, and A W that part of the force P, which tends to give motion to the particle along the plane.

Put $a$ for the angle, P W C, then,

Pressure on the plane = C W = P *cos.* $a$; and the resistance of friction

$$= f \; P \; cos. \; a;$$

The effective pressure tending to move the particle in opposition to friction

$$= A \; W = P \; sin. \; a;$$

Motion will or will not take place, according as their pressure is greater or less than the resistance of friction;

and when motion is upon the point of taking place, the one must be equal to the other, and then the angle $a$, is called the limiting angle of resistance ;

Therefore in this case

$$f\ P\ cos.\ a = P\ sin.\ a$$

$$\therefore\ f = \frac{sin.\ a}{cos.\ a} = tan.\ a.$$

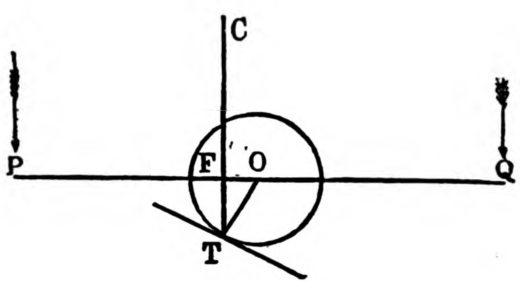
This equation shows that the co-efficient of friction is equal to the tangent of the limiting angle of resistance. If the pressure be applied within the angle, then no motion can take place, however great that pressure may be ; and on the contrary, if the pressure be applied without this angle, then motion will take place, however small that pressure may be. Foreign writers make this property the basis of their theory of machines.

*Ques.* 36. The co-efficient of friction, $f = .683$, what is the limiting angle of resistance ?

From a table of natural tangents it will be found, that the angle 34°..20′ has a natural tangent nearly equal .683 ; hence, the limiting angle of resistance = 34°..20′.

*Ques.* 37. *Equilibrium of the lever, and the wheel and axle, taking the friction upon the axis into account ?*

Let P O Q be a lever, having the circular axis C T, turning within a circular socket. In order that motion should




take place in the direction of the pressure P, the resultant of the two pressures P and Q must fall on the left side of the centre O of the axis, and by the last problem, this re-

sultant C T, must be inclined to O T at the limiting angle of resistance.

Taking T, therefore, as the centre of motion, by the equality of moments for the state bordering on motion,

$$P \times P F = Q \times Q F.$$

*Ques.* 38. Let P O = 14 inches ; Q O = 12 inches, P = 27 lbs., and angle O T F = 30°, O T = 3 inches ?

The natural sine of 30° = .500000.*

$$O F = \textit{sin. a}\ 30° \times 3 = 1.5 ;$$

$$P F = 14 - 1.5 = 12.5 ; \quad Q F = 12 + 1.5 = 13.5 ;$$

$$\therefore \quad 27 \times 12.5 = Q \times 13.5$$

$$\therefore \quad Q = 25 \text{ lbs.}$$

*Ques.* 39. Required the same as in the last example, when the angle O T F = 40°, and P = 100 lbs. ?

Natural sine of 40° = .6427876 ;

.643 will be near enough for practical purposes.

$$O F = \textit{sin.}\ 40 \times 3 = .643 \times 3 = 1.929 ;$$

$$P F = 14 - 1.929 = 12.071 ; \quad Q F = 12 + 1.929 = 13.929 ;$$

$$\therefore \quad 100 \times 12.071 = Q \times 13.929 ;$$

$$\therefore \quad Q = 86.6 \text{ lbs.}$$

*Ques.* 40. Let O P be the radius of a wheel, and O Q the radius of an axle ; put W for the weight of the wheel and axle, then as this weight acts through the centre O, then,

$$P \times P F = Q \times Q F + W \times O F ;$$

Suppose the radius of the = 20 inches, the axle 4 inches, and the axis 2 inches ; required Q, when the power applied to the wheel = 100 lbs., the weight of the wheel and axle = 80 lbs., and the limiting angle of resistance = 21°.

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* See the "*Practical Model Calculator*," by Oliver Byrne, the author of the present work.

The natural sine of 21° = .3583679

.358 is near enough in practice.

$$O\,F = \textit{sin.}\ 21° \times 2 = .716\ ;$$

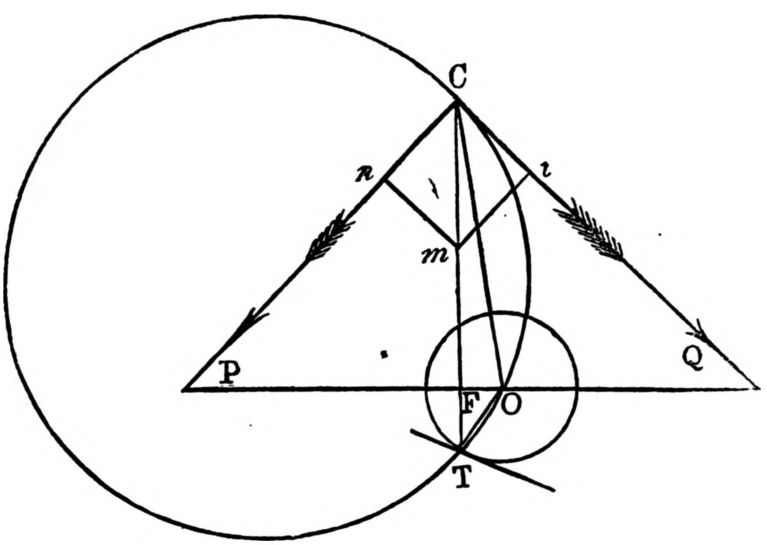
$$P\,F = 20 - .716 = 19.284\ ;\quad O\,F = 4.716\ ;$$

$$P = 100\ ;\quad W = 80\ ;$$

$$\therefore\ 100 \times 19.284 = Q \times 4.716 + 80 \times .716$$

$$\therefore\ Q = 396.7 \text{ lbs.}$$

*Ques.* 41. Let the pressures P and Q be inclined to the lever, then produce them till they intersect at C, and join the points C and O ; now the resultant of the forces P and Q must be drawn, so that the angle O T C = the limiting angle of resistance ; for this purpose, describe a circle upon the chord C O, so as to contain the limiting angle of resistance ?




Let this circle intersect the circumference of the axis at the point T, join T and C ; then on the line C P take off a space C *n*, equal to the units in the pressure P, and construct the parallelogram of pressures C *n m t*, then the units in C *t* will give the pressure of Q, when the motion is upon the point of taking place.

Values of $f$, or tan. $a$, according to the experiments of M. Morin.

| | |
|---|---|
| Iron on oak | .62 |
| Cast iron on oak | .49 |
| Oak on oak, fibres parallel | .48 |
| Oak on oak, greased | .10 |
| Cast iron on cast iron | .15 |
| Wrought iron on wrought iron | .15 |
| Brass on iron | .16 |
| Brass on brass | .20 |
| Wrought iron on cast iron | .19 |
| Cast iron on elm | .19 |
| Soft limestone on the same | .64 |
| Hard limestone on the same | .38 |
| Leather belts on wooden pulleys | .47 |
| Cast iron on cast iron, greased | .10 |
| Brass on iron, greased | .08 |
| Pivots, or Axis of wrought iron, or cast iron, or brass, or cast iron pillow blocks ; | |
| Constantly supplied with oil | .05 |
| Greased from time to time | .08 |
| Without any application, or dry | .15 |

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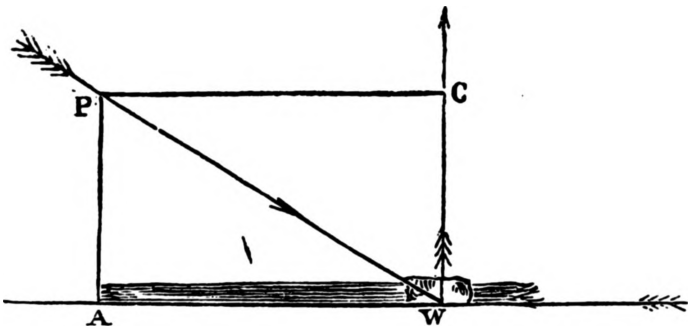
## TO DETERMINE GENERALLY THE FORCE OF TRACTION.

LET W be a heavy body drawn alone the horizontal plane A W, by the force of the traction P, acting in the direc-




tion and with the magnitude W P. This force into W A, and W C, the former force just overcomes the resistance

of friction, the latter tends to reduce the pressure of the body on the plane; hence, putting $b$ = angle P W A.

Pressure of W on the plane = W — P *sin.* $b$.

$\therefore$ the resistance of friction

$= f$ (W — P *sin.* $b$.)

But when motion is about to take place, this resistance is equal to the force W A,

or = P *cos.* $b$;

$\therefore$ P *cos.* $b = f$ W — *sin.* $b$);

$$\therefore \quad P = \frac{f\ (W)}{cos.\ b + f\ sin.\ b}; \quad (z).$$

In the same manner, the traction may be found when the body is moved upon an inclined plane.

*Ques.* 42. A stone weighing 2 cwt., is drawn along a horizontal plane, by a cord inclined at an angle of 35°; required the force necessary to move the stone, supposing the co-efficient of friction to be = .70?

Natural sine of 35° = .574
Natural cosine of 35° = .809 } nearly.

$$P = \frac{.7 \times 2.}{.809 + .7 \times .574} = 1.15 \text{ cwts.}$$

TO FIND THE LEAST TRACTION.

In equation (Z), substitute the value of $f$ before given

$$= \frac{sin.\ a}{cos.\ a};$$

then by an easy reduction, it will be found that,

$$P = \frac{W\ sin.\ a}{cos.\ (b - a)};$$

It is evident that P will be the least possible, when the denominator of this expression is the greatest possible, which will obviously happen when

$$b - a = o,$$

$$\text{or } b = a;$$

that is, when the angle of traction is equal to the limiting angle of resistance.

Therefore, in this case,

$$P = W \sin. a; \quad (J).$$

*Ques.* 43. A stone weighs 10 cwt., and is drawn along a horizontal plane; required the least traction, taking $a = 23°$;

By equation (J); Nat. *sin.* $23° = .3907$

$$P = 10 \times .3907 = 3.907 \text{ cwts.}$$

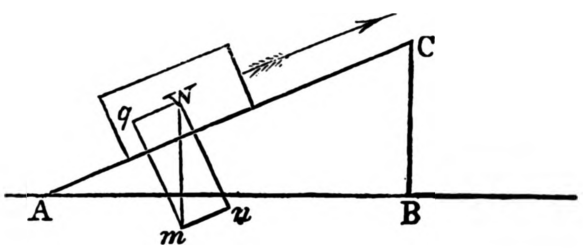
*Ques.* 44. Required the least traction, when the weight of the mass is 9 cwt., and the rubbing surfaces are soft calcareous stone upon the same, and the limiting angle of resistance $= 36°..30'$.

Natural sine of $36°..30' = .5948$

$$9$$

$$\text{Least traction} = 5.3532 \text{ cwt.}$$

*Ques.* 45. If a heavy body be moved upon a rubbing surface, lying between two given points, by a pressure acting parallel to the surface, the work is always the same, whatever may be the form of the surface.




Let W be a body moved on the plane A C, then by the resolution of pressures, the pressure of W on the plane $=$ W *cos.* A; W *q m n*, being the parallelogram of pressures, and since W *n* is perpendicular to A C, and *m n* parallel

to it, the angle $m$ W $n$ = the angle A ; therefore, the resistance of friction

$$= f \text{ W } cos. \text{ A ;}$$

Consequently, the total work on the plane A C

$$= f \ W \ cos. \ A \times A \ C + W \times C \ B$$
$$= f \ W \times A \ B + W \times C \ B.$$

This final expression is *independent of the inclination of the plane*, it being, in fact, the work expended in moving the body over the horizontal distance A B, added to the work due to gravity in elevating the body to the vertical space B C.

As a curved surface may be regarded as being made up of an infinite number of straight planes, therefore, the work upon the whole curve will be equal to the work done upon the horizontal projection of the curve, added to the work done in opposition to gravity. When bodies descend a surface, the work of gravity becomes minus.

This total work is obviously, independent of the nature of the curve. In general, it may be shown, that whatever may be the zigzag course pursued by the body, the work will always be equal to the work on the projection of this curve, added to the work due to gravity. When the inclination of the plane is small, the horizontal distance may be taken equal to the length of the plane. See Chapters I. and II.

*Ques.* 46. What work would a horse be able to perform in drawing a load of one ton to the HORIZONTAL distance of two miles, up a curved incline, whose total elevation = 500 feet, the co-efficient of friction being = $\frac{1}{32}$ ?

Work on the horizontal line

$$= \frac{2240}{32} \times 2 \times 5280 = 739200 ;$$

Work due to gravity

$$= 2240 \times 500 = 1120000 ;$$

Total work = $739200 + 1120000 = 1859200$.

If the body W, be upon the point of descending the plane by the force of gravity alone, then, the work necessary to move the body is nothing; hence,

$$f\ W \times A\ B - W \times C\ B = O$$

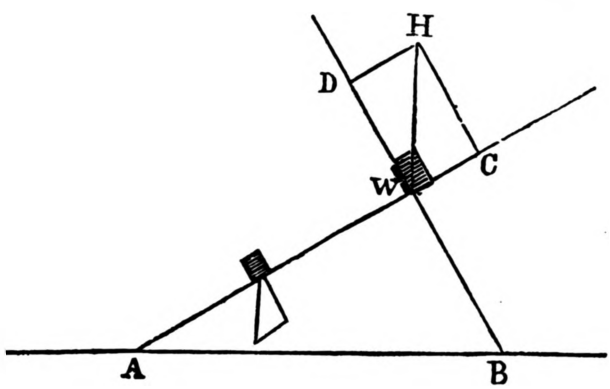
$$\therefore\ f = tan.\ A\ ;$$

a result before obtained.

---

## THE PARALLELOGRAM OF PRESSURES PROVED ON THE PRINCIPLE OF WORK.

LET B D, and A C, be two inclined planes intersecting each other at right angles in the point W. Let a body W, be placed between the two planes; then the force with




which the body tends to descend the plane D B, will be equal to the pressure which it exerts upon the plane C A; and the tendency to descend the plane C A will be equal to the pressure exerted on the plane D B.

Let the vertical line W H be drawn through the body, and take W H = the units of weight in W; from H let fall H C, perpendicular to the plane A C, and H D perpendicular to the plane B D, then putting A and B for the inclinations of the planes A C and B D, by trigonometry

$$W\ C = w \times sin.\ A =$$

the tendency down the plane A C.

Similarly, W D = *w sin.* B = the tendency down the plane B D. But these forces W C and W D, are generated by the gravity of the body, or a force represented by the diagonal W H. Having thus established the proposition for the case of the rectangle, it may be very readily extended to the general form of the parallelogram.

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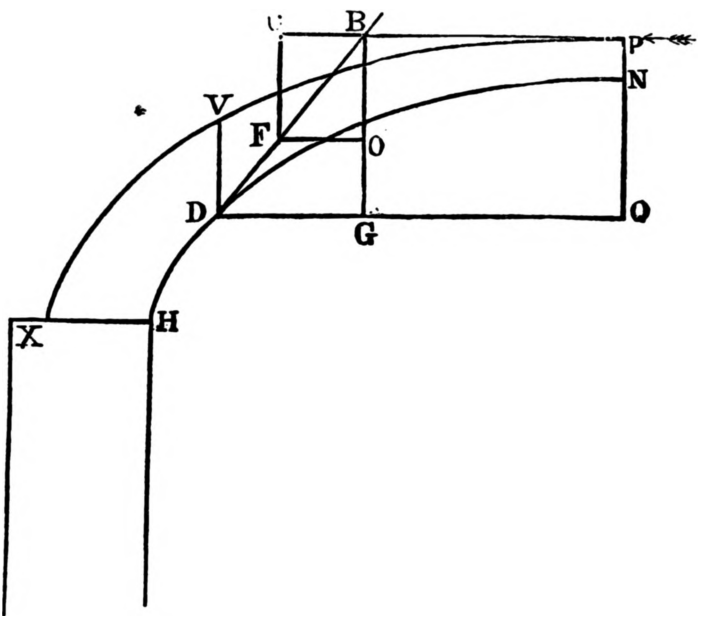
## THE EQUILIBRIUM OF THE ARCH.

WHEN the centres of an arc are taken away, the crown almost invariably sinks; this occasions the joint at the crown to open at its lower edge, and at the same time a certain portion, D V P N, of the arc to turn upon D as a centre, thereby producing a rupture, or opening, in the exterior edge at this point. The same effect will take place in the other half of the arch.




These two equal portions which thus tend to break away from the general mass, exert a horizontal pressure along the line P C, thereby tending to cause the walls of the structure to turn on their outer edges. The arc will undergo a rupture at that point where the portion D V P N, so breaking away, will produce the greatest horizontal thrust;

for this point must, obviously, be the yielding point of the arc.

*To find the point of Rupture.*

Let B G, be a vertical line, passing through the centre of gravity of the mass D V P N, and intersecting P C in the point B ; join the points B and D, take O B equal to the units of weight in the mass D V P N, and construct the rectangle O F C B, then the units in B C will be the horizontal thrust, if it be greater than any other so determined, and the point D will be the point of rupture. MM. Clapeyron and Lame, found that the resultant D B forms a tangent to the intrados of the arch, when the line of rupture is assumed to follow the vertical line D V. This property gives a very easy method for finding the point of rupture ; for if upon constructing the figure, as already described, it is found that B D touches the curve, then D will be the point of rupture. The most troublesome part of this calculation, consists in finding the centre of gravity of the mass D V P N. However, the following method will be found sufficiently accurate for practical purposes :

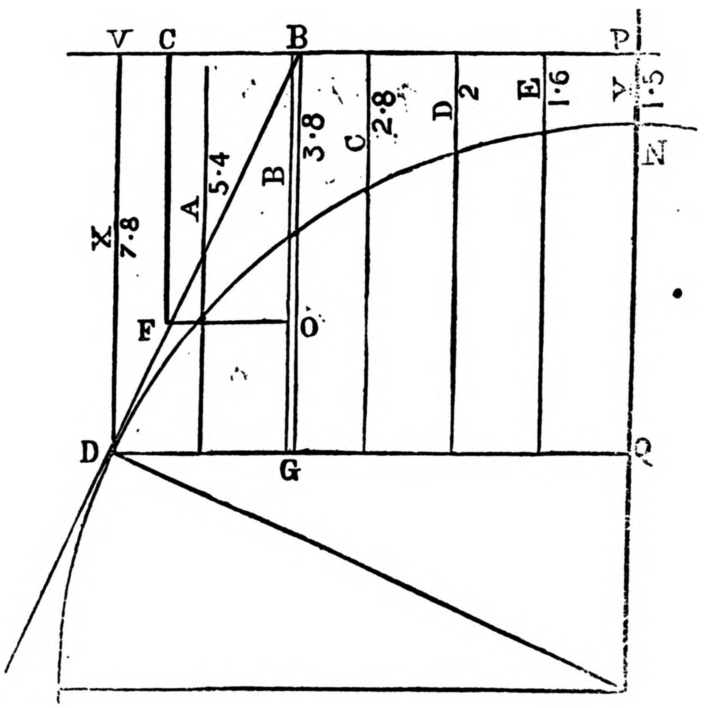
If D Q be divided into $n$ equal parts, and lines be drawn from the points of division perpendicular to D Q, cutting the intrados and extrados ; let $x$ and $y$ be put for D V and N P, the extreme ordinates respectively, and A, B, C, &c., for the first, second, third, &c., intermediate ordinates, or vertical distances between the intrados and extrados, then putting $s$ = the common distance between the ordinates

$$\mathrm{D\,G} = \frac{\mathrm{S}\left\{\mathrm{X} + (3n-1)\,\mathrm{Y} + 6\,(\mathrm{A} + 2\,\mathrm{B} + 3\,\mathrm{C} + \&c.)\right\}}{3\left\{\mathrm{X} + \mathrm{Y} + 2\,(\mathrm{A} + \mathrm{B} + \mathrm{C} + \&c.)\right\}}.$$

If that part of the divisor which is within the brackets be multipled by $\frac{s}{2}$ it will give the area of D V P N.

Having ascertained the horizontal thrust, B C, the centre of gravity of the whole mass, including the semi-arch X P N H, with its load, if any, and the pier H X ; then supposing the pier to turn upon its outer edge, if the moment of this mass exceeds the moment of the horizontal thrust, the structure will stand, and *vice versâ*.

*Ques.* 47. The radius of a semicircular arc is 11 feet, the thickness of the crown 18 inches ; the mason work is built level with the crown, and the weight of a cubic foot of the material = 120 lbs. ; required the point of rupture, and the horizontal thrust, taking a depth of 1 foot of the masonry ?




Construct a figure on a scale of an inch to the foot, and dividing D Q into six equal parts, it may be found by calculation or measurement, that X = 7.8, A = 5.4, B = 3.8, C = 2.8, D = 2, E = 1.6, Y = 1.5, and $n$ = 6, and D Q = 10, and hence $s = \frac{1}{6}^{0}$.

By the formula it will be found, that D G = 3.53 feet, and the weight D V P N = 4000 lbs. Take O B = 4000, and construct the parallelogram O C, then B C = 1900 lbs. nearly ; and as B D forms a tangent to the curve, the point D will be the point of rupture. The point D is 65° from N, the crown.

*Ques.* 48. If the piers in the last example be 4 feet thick, and their height 28.5 feet measured to the level of the crown ; will they stand or fall ?

In this case, tne weight of the whole semi-arch = 5100 lbs., taking as before the depth, one foot.

The distance of the centre of gravity from the outer edge of the pier = 7.6 feet. Weight of the whole pier = $4 \times 1 \times 28.5 \times 120 = 13680$ lbs.

Hence the moment of the pier

$$= 13680 \times 2 = 27360 ;$$

The moment of the semi-arch

$$= 5100 \times 7.6 = 38760 ;$$

and the moment of the horizontal thrust

$$= 1900 \times 28.5 = 54150 ;$$

The sum of the two first moments being greater than the moment of the thrust, it follows that the pier will stand. But it will be found in the same manner, if the pier be only three feet thick the structure will fall. When the pier is 3.3 feet thick it will be on the point of overturning.

## CHAPTER VIII.

### WOODEN AND IRON BRIDGES.

THE equilibrium of wooden and iron bridges will be considered after I give some investigations, illustrating a higher method of investigating mechanical subjects.

Variable motion is caused by a force, which, acting continually upon a moving body, goes on at every instant increasing or diminishing its velocity. Such force is measured at each instant, by the ratio of that very small increment or decrement of velocity, which it causes in the moving body, to that very small time in which the increment or decrement is caused. When this ratio is constant, or so that in equal times equal degrees of velocity are added to, or taken from the moving body, the accelerating or retarding

force is constant, but when this ratio is variable, the force is also variable.

It is more convenient to use the symbol $\lceil\overline{\ }$ instead of the usual symbol $d$, employed in the calculus; $\lceil\overline{\ }$ must not be taken for $\surd$, which is the well-known sign of the square root.

Instead of $dy$, $dx$, &c., I write

$$\lceil\overline{y}, \quad \lceil\overline{x}, \quad \&c.$$

In the case of a variable motion, Put F for the accelerating force, $V$ the velocity, $S$ the space, and $T$ the time, the relations between these qualities may be found by the equations

$$V = \frac{\lceil\overline{S}}{\lceil\overline{T}}, \qquad \text{(I)};$$

$$F = \frac{\lceil\overline{N}}{\lceil\overline{T}}, \qquad \text{(II)}.$$

Other general forms may be deduced from these, for example, from (I),

$$\lceil\overline{T} = \frac{\lceil\overline{S}}{\lceil\overline{V}}, \text{ and from (II)},$$

$$\lceil\overline{T} = \frac{\lceil\overline{V}}{\lceil\overline{F}},$$

$$\therefore \quad \frac{\lceil\overline{S}}{V} = \frac{\lceil\overline{V}}{F}, \text{ and}$$

$$\therefore \quad F\lceil\overline{S} = V\lceil\overline{V}.$$

Again eliminating V from (I) and (II),

$$F\lceil\overline{T} = \lceil\overline{V} = \lceil\frac{\lceil\overline{S}}{\lceil\overline{T}};$$

$$\therefore \quad F\lceil\overline{T} = \lceil\frac{\lceil\overline{S}}{\lceil\overline{T}}.$$

On the supposition that $T$ is the independent variable, the last equation becomes

$$F = \frac{\overline{|S}^{2}}{\overline{|T}^{2}}.$$

The above equations relate to accelerated motions, by making $|\overline{V}$ negative, they are adopted to retarded motions.

In an equally accelerated motion $F$, is constant, from (II) therefore,

$$\int F|\overline{T} = \int |\overline{V}, \text{ integrated}$$

$$\text{becomes} \quad F\,T = \quad V; \qquad \text{(III)}.$$

The velocities, therefore increase as the time when the force is constant. It is not necessary to correct (III), when the body moves from a state of rest, for then when $T = o$, $V = o$, and $F\,T = o$. But if the body has a velocity $a$, already impressed, then (III) requires correction and becomes

$$\mathrm{V} = a + F\,T, \qquad \text{(IV)};$$

for V, becomes $a$, in (IV), as it should do, when $T = o$.

When the force is retarding, V becomes negative and (IV) becomes

$$\mathrm{V} = a - F\,T$$

It must not be forgotten that $\overline{\ }$ stands for the symbol $d$.

Again integrating $F|\overline{S} = V|\overline{V}$,

$$F\int |\overline{S} = \int V|\overline{V},$$

$$\therefore \quad F\,S = \tfrac{1}{2}\,\mathrm{V}^2$$

$$\therefore \quad S = \frac{\mathrm{V}^2}{2\,F} + \text{constant.} \qquad \text{(V)}.$$

If the body moves from a state of rest (V) requires no correction, or addition of a constant, for in this case $S$ must be equal to 0, when $V = 0$. But if the body should have a velocity $a$, before the force $F$ commences to act, then the space passed over in the time $T$ will be

$$S = \frac{\mathrm{V}^2}{2\,F} - \frac{a^2}{2\,F},$$

$$= \frac{V^2 - a^2}{2F},$$

because, when $T = 0$, the body had a velocity $a$, and hence must have passed over a space

$$= \frac{a\,2}{2F}, \text{ before,}$$

$$S = \frac{V^2 - a^2}{2F}, \qquad \text{(VI),}$$

commences to be described.

Equation (VI), is readily obtained by integrating and correcting the general equation $F \lceil S = V \lceil V$, the result in this case is the equation

$$S = \frac{a^2 - V^2}{2F}.$$

Another sent of general expression may be thus formed,—From (II),

$$F \lceil T = \lceil V, \text{ integrating and}$$

$$F\,T = V = \frac{\lceil S}{\lceil T}, \text{ according to (I),}$$

$$\therefore \quad \lceil S = F\,T \lceil T$$

integrating, and the result is

$$S = \frac{F\,T^2}{2} + \text{constant}; \qquad \text{(VII.)}$$

If the body move from a state of rest, (VII) requires no correction, for $S$ and $T$ vanish simultaneously, in this case also, by transposing

$$F = \frac{2\,S}{T^2},$$

and consequently the measure of the constant accelerating force $F$ is obtained, when the space, which the moving body has described in a given time $T$, is known. Near the surface of the earth gravity may be considered a constant

force, and since a heavy body falling from rest describes $16\frac{1}{12}$ feet in one second

$$F = 32\frac{1}{6} \text{ feet,}$$

which is generally represented by the letter $g$, by most writers and mechanics.

It must be remembered that $\lceil\!\!\overline{\ }$ stands for $d$ of the Calculus, and must not be taken for $\surd$, the sign of the square root.

Correcting (VII) for accelerated and retarded motion, with a constant velocity $a$, before the force $F$ commences to act,

$$S = a\ T + \frac{F\ T^2}{2}, \text{ (accelerated.)}$$

$$S = a\ T - \frac{F\ T^2}{2}, \text{ (retarded.)}$$

The reasons for these last expressions are obvious, for supposing the force $F$ to cease to act during the time $T$ then (VII) becomes

$$S = o + \text{constant} = a\ T$$

the uniform velocity $a$ multiplied by the time $T$.

---

## THE VERTICAL MOTIONS OF HEAVY BODIES THROUGH RESISTING MEDIUMS.

For want of sufficient experiments, Mathematicians take for granted, that the air and other fluids offer resistances proportional to the squre of the velocity of the body moving in them.

Let $v$ be the velocity, then this retarding force may be expressed by $v^2$ multiplied by a constant co-efficient, A, so that the retarding force is $A\ v^2$, the value of A will depend on the figure of the body, and upon the ratio of its specific gravity to that of the fluid. Consequently, if a body descend from a state of rest through a resisting medium, the

accelerating force is $g - A v^2$, which put for F in (I) and (II) gives, putting $t$ for the time, and $s$ the space,

$$(g - A v^2) \lceil \overline{t} = \lceil \overline{v}, \text{ and}$$
$$(g - A v^2) \lceil \overline{s} = \lceil \overline{v}.$$

For the convenience of calculation put $m =$ the square root of $g$, and $n =$ the square root of $a$; then $A = g n^2$, then the two last equations become

$$g \lceil \overline{t} = \frac{\lceil \overline{v}}{1 - n^2 v^2}; \quad g \lceil \overline{s} = \frac{v \lceil \overline{v}}{1 - n^2 v^2}.$$

There will result the two following equations by integration and correcting the results on the supposition, that when $t = o$, $s = o$, and $v = o$;

$$v = \frac{1}{n} \frac{\text{E } 2 g n t - 1}{\text{E } 2 g n t + 1} \text{ and } s = \frac{1}{2 g n t^2} \log \frac{1}{1 - n^2 v^2};$$

a third equation may be found by eliminating $v$,

$$s = \frac{1}{g n^2} \log \tfrac{1}{2} \left\{ \frac{g n t}{\text{E}} + \frac{-g n t}{\text{E}} \right\}$$

---

## CURVILINEAR MOTION.

Suppose the position of the moveable point to be referred to three coördinate axes, and let the acting forces be reduced to three, P, Q, R, parallel to these coördinates.

Then $\lceil \overline{s}$ being the small arc which the point will describe in the time $\lceil \overline{t}$, its velocity at this point will be

$$v = \frac{\lceil \overline{s}}{\lceil \overline{t}},$$

because $\lceil \overline{s}$ is the diagonal of a parallelopiped, which has for its sides $\lceil \overline{x}$, $\lceil \overline{y}$, $\lceil \overline{z}$, the moving point cannot, in the instant $\lceil \overline{t}$, describe the small arc $\lceil \overline{s}$, without at the same

time describing, in the directions of the ordinates of $x$, $y$, $z$, the small spaces $\lceil x$, $\lceil y$, $\lceil z$; for instead of the velocity $\frac{\lceil s}{\lceil t}$, the point may be imagined to move with the velocities

$$\frac{\lceil x}{\lceil t}, \quad \frac{\lceil y}{\lceil t}, \quad \frac{\lceil z}{\lceil t},$$

parallel to the ordinates of $x$, $y$, $z$, hence the equation

$$F \lceil T = \lceil \frac{\lceil S}{\lceil T};$$

derived from (I) and (II), gives three following equations,

$$P \lceil t = \lceil \frac{\lceil x}{\lceil t}; \quad Q \lceil t = \lceil \frac{\lceil y}{\lceil t}; \quad R \lceil t = \lceil \frac{\lceil z}{\lceil t}.$$

Integrating these equations, the partial velocities

$$\frac{\lceil x}{\lceil t}, \quad \frac{\lceil y}{\lceil t}, \quad \frac{\lceil z}{\lceil t}$$

are obtained, from the composition of which results the absolute velocity of the point. Integrating again, and the ordinates $x$, $y$, $z$, are found in terms of $t$, and hence the place of the point at any instant. When $t$ is eliminated, there will remain two equations between $x$, $y$, $z$, the equations of the curve described by the moving point.

The student must keep in mind that $\lceil$ stands for the differential symbol $d$.

When the forces acting on the moving point are all in the same plane, the trajectory will lie wholly in that plane, and its equation will be found by eliminating $t$, from the two equations.

$$P \lceil t = \lceil \frac{\lceil x}{\lceil t} \text{ and } Q \lceil t = \lceil \frac{\lceil y}{\lceil t};$$

which may be written

$$P = \frac{\lceil^2 x}{\lceil t^2} \text{ and } Q = \frac{\lceil^2 y}{\lceil t^2}; \qquad \text{(VIII.)}$$

$t$ being considered the independent variable, and $\lceil^2$ is put

instead of $d^2$ and, $\lceil\overline{\ }^2$ instead of $(dx)^2$, which is written $dx^2$.

Supposing the ordinates to be rectangular, then

$$\lceil\overline{x}^2 + \lceil\overline{y}^2 + \lceil\overline{z}^2 = \lceil\overline{s}^2$$

$$\therefore\ 2\lceil\overline{x}\ \overset{2}{\lceil\overline{x}} + 2\lceil\overline{y}\ \overset{2}{\lceil\overline{y}} + 2\lceil\overline{z}\ \overset{2}{\lceil\overline{z}} = 2\lceil\overline{s}\ \overset{2}{\lceil\overline{s}}.$$

Multiply (VIII) by $\lceil\overline{x}$, $\lceil\overline{y}$, &c., and add together,

$$P\lceil\overline{x} + Q\lceil\overline{y} + R\lceil\overline{z} = \frac{\lceil\overline{x}\ \overset{2}{\lceil\overline{x}} + \lceil\overline{y}\ \overset{2}{\lceil\overline{y}} + \lceil\overline{z}\ \overset{2}{\lceil\overline{z}}}{\lceil\overline{t}^2}.$$

$$\therefore\ P\lceil\overline{x} + Q\lceil\overline{y} + R\lceil\overline{z} = \frac{\lceil\overline{s}\ \overset{2}{\lceil\overline{s}}}{\lceil\overline{t}^2}.$$

$$\therefore\ P\lceil\overline{x} + Q\lceil\overline{y} + R\lceil\overline{z} = v\lceil\overline{v}, \qquad \text{(IX.)}$$

because,

$$v = \frac{\lceil\overline{s}}{\lceil\overline{t}} \text{ and}$$

$$\therefore\ \lceil\overline{v} = \frac{\overset{2}{\lceil\overline{s}}}{\lceil\overline{t}},$$

$$\therefore\ v\lceil\overline{v} = \frac{\lceil\overline{s}\ \overset{2}{\lceil\overline{s}}}{\lceil\overline{t}^2}.$$

*On the motion of Cannon Shot.*

If a body be projected obliquely, with a velocity due to the altitude $h$, (the resistance of the air being neglected), and if the abscissa of $x$ be taken in the vertical line drawn through the point of projection, and the ordinates of $y$ parallel to the direction of projection, the equation of the curve described by the projectile will be

$$y^2 = 4hx.$$

Taking the general equations (VIII,)

$$P \sqrt{t} = \int \frac{\sqrt{x}}{\sqrt{t}} \text{ and } Q \sqrt{t} = \int \frac{\sqrt{y}}{\sqrt{t}},$$

putting $P = g$ and $Q = o$, and integrating, then

$$\frac{\sqrt{x}}{\sqrt{t}} = g\,t + \text{constant}\,; \quad \frac{\sqrt{y}}{\sqrt{t}} = \text{constant}.$$

To find the constant quantities, it must be observed that when $t = o$, the velocity in the direction of $x = o$,

$$\therefore \quad \frac{\sqrt{x}}{\sqrt{t}}\, o + \text{constant} = o, \text{ hence constant } o\,;$$

$$\therefore \quad \frac{\sqrt{x}}{\sqrt{t}} = g\,t\,; \qquad \text{(X.)}$$

requires no correction. Again, when $t = o$, the velocity in the direction of $y$ is that due to the height $h$, which is

$$= (2\,g\,h)^{\frac{1}{2}},$$

$$\therefore \quad \frac{\sqrt{y}}{\sqrt{t}} = \text{constant} = (2\,g\,h)^{\frac{1}{2}}, \text{ the required constant.}$$

$$\therefore \quad \frac{\sqrt{y}}{\sqrt{t}} = (2\,g\,h)^{\frac{1}{2}}, \qquad \text{(XI.)}$$

Integrate (X) and (XI), and consider $t$, as before, the independent variable, then

$$x = \tfrac{1}{2}\,g\,t^2 \text{ and } y = (2\,g\,h)^{\frac{1}{2}}\,t,$$

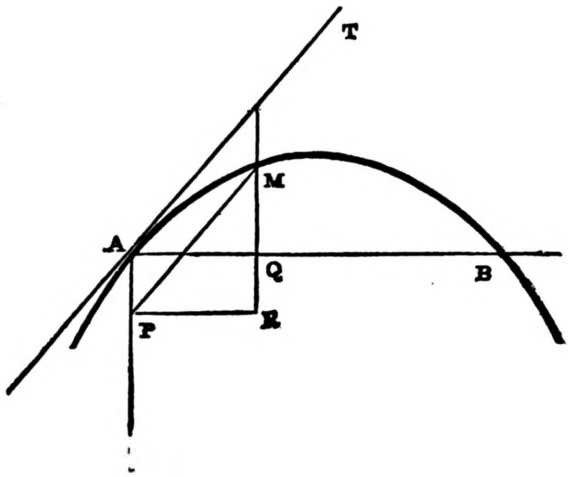
these results require no correction, for when $t = o$, $x = o$ and $y = o$, a condition which the resulting equations satisfy without the aid of a constant quantity. Eliminate $t$, then

$$y^2 = 4\,h\,x, \qquad \text{(XII.)}$$

an equation to the common parabola, having for its diameter the vertical drawn through the point of projection; and the parameter is 4 times the height due to the initial velocity of the projection.

Let the coördinates be changed, and instead of taking A P for the abscissa of $x$, and P M for the ordinate of $y$, assume the horizontal line A Q = $x$, and the vertical Q M

$= y_1$ ; let the angle of elevation T A B $= \theta$, and draw P R parallel to A B.




$$\text{M R} = \text{P M } \sin.\ \theta\text{; P R} = \text{P M } \cos.\ \theta\text{;}$$

$$\text{or } x_1 + x = y \sin.\ \theta\text{; } y_1 = y \cos.\ \theta.$$

$$\therefore\ x_1 = y_1 \tan.\ \theta - x$$

$$\text{But (XII), } x = \frac{y^2}{4h} = \frac{y_1^2}{4h \cos.^2 \theta}.$$

$$\therefore\ x_1 = y_1 \tan.\ \theta - \frac{y_1^2}{4h \cos.^2 \theta}; \qquad \text{(XIII.)}$$

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### THE PATH OF A PROJECTILE IN THE AIR.

LET $g\,r$ express the resistance of the medium. This force being in direction of $\lceil\overline{s}$ ; now, if the curve be referred to an horizontal axis as abscissas of $x$, and the ordinates $y$ vertical, the resistance $g\,r$ may be resolved into two forces

$$g\,r\,\frac{\lceil\overline{x}}{\lceil\overline{s}} \text{ and } g\,r\,\frac{\lceil\overline{y}}{\lceil\overline{s}},$$

one parallel to the axis of $x$ and the other to that of $y$. Consequently, the body will be acted upon in the direction

of $x$ by the force $-gr\frac{\sqrt{x}}{\sqrt{s}}$ and at the same time in the direction of $y$ by the force $-g-gr\frac{\sqrt{y}}{\sqrt{s}}$. Then the general equations (VIII,) are easily applied, they become

$$-gr\frac{\sqrt{x}}{\sqrt{s}}\sqrt{t} = \int\frac{\sqrt{x}}{\sqrt{t}}, \qquad \text{(XIV.)}$$

$$-g\sqrt{t}-gr\frac{\sqrt{y}}{\sqrt{s}}\sqrt{t} = \int\frac{\sqrt{y}}{\sqrt{t}}, \qquad \text{(XV.)}$$

From these equations $t$ is to be eliminated. On the supposition that $\sqrt{x}$ is constant

$$\int\frac{\sqrt{x}}{\sqrt{t}} = \int\sqrt{x}\sqrt{t}^{-1} = \sqrt{x}\int\sqrt{t}^{-1}$$

$$= -\sqrt{x}\sqrt{t}^{-2}\sqrt{t}^{2} = -\frac{\sqrt{x}\sqrt{t}^{2}}{\sqrt{t}^{2}}$$

$$\therefore\ -gr\frac{\sqrt{x}}{\sqrt{s}}\sqrt{t} = -\frac{\sqrt{x}\sqrt{t}^{2}}{\sqrt{t}^{2}}$$

$$\therefore\ gr\sqrt{t}^{3} = \sqrt{s}\sqrt{t}^{2}, \qquad \text{(XVI.)}$$

In works on the calculus $\sqrt{t}^{3}$ is written $dt^{3}$, and $\sqrt{t}^{2}$ is written $d^{2}t$, the symbol $d$ is always represented by $\sqrt{\ }$, and is not to be taken for $\surd$, the sign of the square root.

From (XVI) $\sqrt{t}^{2} = \frac{gr\sqrt{t}^{2}}{\sqrt{s}}$,

$$\int\frac{\sqrt{y}}{\sqrt{t}} = \frac{\sqrt{y}^{2}}{\sqrt{t}} - \frac{\sqrt{y}\sqrt{t}^{2}}{\sqrt{t}^{2}}$$

$$= \frac{\sqrt{y}^{2}}{\sqrt{t.}} - \frac{\sqrt{y}}{\sqrt{+}^{2}}gr\frac{\sqrt{+}^{3}}{\sqrt{s}}$$

$$= \frac{\sqrt{y}^{2}}{\sqrt{t}} - gr\frac{\sqrt{y}}{\sqrt{s}}\sqrt{t}$$

Consequently, (XV) becomes

$$-g\sqrt{t^2} = \sqrt[2]{y}, \qquad \text{(XVII.)}$$

From (XVII) another equation is readily found,

$$\sqrt[3]{y} = 2g\sqrt{t}\ \sqrt[2]{t}, \qquad \text{(XVIII.)}$$

$\sqrt{t}$, $\sqrt[2]{t}$, are easily eliminated from equations (XVI), (XVII), (XVIII), hence

$$2r\sqrt[2]{y}^2 + \sqrt{s}\ \sqrt[3]{y} = o, \qquad \text{(XIX)};$$

which is the equation of the trajectory; when the laws of resistance are determined, the curve described by the projectile becomes known from (XIX.)

Suppose the resistance to be proportional to the square of the velocity, then put $gr = gn^2v^2 = gn^2\frac{\sqrt{s}^2}{\sqrt{t}^2}$.

But from (XVII)

$$\sqrt{t}^2 = -\frac{\sqrt[2]{y}}{g}$$

$$\therefore\quad r = -gn^2\frac{\sqrt{s}^2}{\sqrt[2]{y}}$$

Hence the equation of the trajectory (XIX), becomes

$$\frac{\sqrt[3]{y}}{\sqrt[2]{y}} = 2gn\sqrt[2]{s}, \qquad \text{(XX.)}$$

It is easily observed that $\int -\frac{\sqrt[2]{y}}{\sqrt{x}^2}$ divided by $\frac{\sqrt[2]{y}}{\sqrt{x}^2}$ gives $\frac{\sqrt[2]{y}}{\sqrt[2]{y}}$, hence, Integrating (XX),

$$\text{Log.}\ \frac{\sqrt[2]{y}}{\sqrt{x}} = 2gn^2s + C$$

$$\text{or, } \frac{\overset{2}{\lceil y}}{\lceil x^2} = C\,E^{2gn^2s}. \qquad \text{(XXI.)}$$

C being the constant required after integrating, and log $c = C$. To find $c$, for $\overset{2}{\lceil y}$ put its value $-g\lceil t^2$, then

$$\frac{\lceil x^2}{\lceil t^2} = -\frac{g}{c\,E^{2gn^2s}}.$$

Now, if $\theta$ be the angle of elevation, and $h$ the height due to the velocity of projection, at the beginning of the motion, or when $s = o$, the horizontal velocity

$$\frac{\lceil x}{\lceil t} = (2gh)^{\frac{1}{2}} \cos.\theta.$$

$$\therefore\ c = \frac{1}{2h\cos.^2\theta}.$$

$\therefore$ (XXI) becomes

$$\frac{\overset{2}{\lceil y}}{\lceil x^2} = -\frac{E^{2gn^2s}}{2h\cos.^2\theta} \qquad \text{(XXII.)}$$

In order to abridge, put $2gn^2 = m$, and $\frac{\lceil y}{\lceil x} = p$, then (XXII) becomes

$$\frac{\lceil p}{\lceil x} = -\frac{E^{ms}}{2h\cos.^2\theta}, \qquad \text{(XXIII.)}$$

Since $\lceil x^2 + \lceil y^2 = \lceil s^2 \quad \therefore\ (1+p^2)^{\frac{1}{2}}\ \lceil x = \lceil s$, and multiplying (XXIII) by this last equation,

$$\lceil p\ (1+p^2)^{\frac{1}{2}} = -\frac{E^{ms}\lceil s}{2h\cos.^2\theta},$$

integration will give the following equation,

$$p(1+p^2)^{\frac{1}{2}} + \log\left[p + (1+p^2)^{\frac{1}{2}}\right] = C_1 - \frac{E^{ms}}{2hm\cos.^2\theta}.$$

Putting P for the left hand member of this equation, and introducing the value of $\frac{\lceil p}{\lceil x}$ from (XXIII), the last equation becomes

$$P = C_1 + \frac{\lceil p}{m \lceil x},$$

$$\therefore \quad \lceil x = \frac{\lceil p}{P m - C_1 m}, \text{ and}$$

$$\lceil y = p \lceil x = \frac{\lceil p}{P m - C_1 m}$$

$$\therefore \quad x = \int \frac{\lceil p}{P m - C_1 m}; \quad y = \int \frac{P \lceil p}{P m - C_1 m}.$$

The constant $C_1$ is found by considering that when $s = o$, $p = tan.\ \theta$. If these formulas could be integrated, $x$ and $y$ would be found in terms of $p$; then eliminating $p$, the equation of the required curve would be found. But since the formulas cannot be integrated in finite terms, the following method of approximation may be employed, when the student cannot apply the Theorems of Laplace and Lagrange.

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## INTEGRATION BY APPROXIMATION.

LET $y = \int X \lceil x$; if when $x = a$, then $y = b$; and if when

$$x = a,\ a + d,\ a + 2d,\ a + 3d \ \ldots\ a + nd;$$

$$\text{then } X = A,\ A_1,\ A_2,\ A_3,\ \ldots\ A n.$$

and if the difference $d$ be very small, the value of $\int X \lceil x$ from

$$x = a \text{ to } x = a + nd,$$

will be,

$$y = b + d\ (\tfrac{1}{2} A + A_1 + A_2\ A_3 \ \ldots\ \tfrac{1}{2} A n).$$

The truth of this is readily established, for in the small interval between $x = a$ and $x = a + d$, it may be assumed that the function X remains constant. And because at the beginning of this interval $X = A$, and at the end $X = A_1$ it may be assumed that this constant value is the arithmetical mean, or $X = \frac{1}{2} A + \frac{1}{2} A_1$.

$\therefore$ For this interval $X \lceil \overline{x} = d (\frac{1}{2} A + \frac{1}{2} A_1)$. For the next interval, comprised between $x = a + d$ and $x = a + 2d$, it may in the same way be shown that

$$X = \tfrac{1}{2} A_1 + \tfrac{1}{2} A_2 ; \quad \therefore \quad X \lceil \overline{x} = d (\tfrac{1}{2} A_1 + \tfrac{1}{2} A_2).$$

Adding this value of $X \lceil \overline{x}$ to the preceding, it appears that the value of $y$ from $x = a$, to $x = a + 2d$, is

$$y = b + d (\tfrac{1}{2} A + A_1 + \tfrac{1}{2} A_2),$$

and so on to the last.

To apply this method to the determination of $x$ and $y$ in this case, it is known when $p = tan.\ \theta$, then $x = o$; first take

$$x = \int \frac{\lceil \overline{p}}{P m - C_1 m},$$

Now since the tangent $p$ of the angle, which the curve makes with the direction of $x$, becomes gradually less and less, until, at the vertex of the curve it is nothing, and afterwards becomes negative, and goes on increasing in the descending branch; make successfully $p = tan.\ \theta$, $tan.\ (\theta - d)$, $tan.\ (\theta - 2d)$, $tan.\ (\theta - 3d)$, &c., until $p = o$; and then $p = -d, -2d. - 3d$, &c., *ad infinitum*; taking for $d$ any number at pleasure; and the smaller it is taken, the more exact will be the discription of the curve. Calculate the values of A, $A_1$, $A_2$, $A_3$, &c., of the quantity

$$\frac{1}{\overline{P m - C_1 m}}$$

when $p$ becomes successively $tan.\ \theta$, $tan.\ (\theta - d)$, $tan.\ (\theta - 2d)$, &c. Then putting successively $n = 1, 2, 3$, &c., *ad infinitum*, the values of $x$ will be found that correspond to the angles where $p = tan.\ (\theta - d)$, $tan.\ (\theta - 2d)$, &c., &c.

Also, by the other equation

$$y = \int \frac{P \sqrt{p}}{P m - C_1 m},$$

the values may be found, which

$$\frac{p}{P m - C m}$$

acquires, when $p$ becomes *tan.* $\theta$, *tan.* $(\theta - d)$, *tan.* $(\theta - 2d)$, &c. ; proceeding in like manner, the values of $y$ will be known, which correspond to the same angles, where $p =$ *tan.* $(\theta - d)$, *tan.* $(\theta - 2d)$, &c.

Hence the abscissas $x$, and the corresponding ordinates $y$ of all the points of the curve, in which $p$ becomes successively *tan.* $(\theta - d)$, *tan.* $(\theta - 2d)$, &c

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## NEW PRACTICAL SOLUTIONS OF IMPORTANT USEFUL PROBLEMS, FROM COMBINING CONTINUED FRACTIONS AND DIOPHANTINE ANALYSIS.

*Question* 1. It can be shown, that it is impossible to represent the perpendicular on one of the sides from the opposite angle of an equilateral triangle in whole numbers, when the sides are expressed in whole numbers ; with this fact in view, let it be required to find a triangle, whose sides and a perpendicular may be expressed in whole numbers, and to have the three sides nearly equal. It is required to have the difference between one of the sides, and either of the other two, less than any fractional part of either of the sides.

Take any two numbers $x$ and $y$, and according to Diophantine Analysis, $2xy$ will be the base, $x^2 - y^2$ the perpendicular, and $x^2 + y^2$ the hypothenuse of a rational right angled triangle. In an equilateral triangle, if the side $=$ 1, the perpendicular $= \frac{1}{2}\sqrt{3}$.

Let $x^2 + y^2 = 1$, and

$$2xy = \tfrac{1}{2}(3)^{\frac{1}{2}}.$$

$$\therefore\ x + y = \left\{1 + \tfrac{1}{2}(3)^{\frac{1}{2}}\right\}^{\frac{1}{2}} = 1.3660254$$

$$x - y = \left\{1 - \tfrac{1}{2}(3)^{\frac{1}{2}}\right\}^{\frac{1}{2}} = 0.3660254$$

$$\therefore\ x = .8660254$$

$$y = .5000000.$$

Now by continued fractions, the nearest whole numbers that will supply the required proportions may be found

| | | | |
|---|---|---|---|
| 8660254 | | 5000000 | |
| 5000000 | (1. *first* | 3660254 | (1. *second* |
| 3660254 | | 1339746 | |
| 2679492 | (2. *third* | 980762 | (1. *fourth* |
| 980762 | | 358984 | |
| 717968 | (2. *fifth* | 262794 | (1. *sixth* |
| 262794 | | 96190 | |
| 192380 | (2. *seventh* | 70414 | (1. *eighth* |
| 70414 | | 25776 | |
| 51552 | (2. *ninth* | 18862 | (1. *tenth* |
| 18862 | | 6914 | |
| 13828 | (2. *eleventh* | 5034 | (1. *twelfth* |
| 5034 | | 1880 | |
| 3760 | (2. *thirteenth* | 1274 | (1. *fourteenth* |
| 1274 | | 606 | |
| 1212 | (2. *fifteenth* | | |

The continued fraction will be

$$1 + \tfrac{1}{1} + \tfrac{1}{2} + \tfrac{1}{1} + \tfrac{1}{2} + \tfrac{1}{1} + \tfrac{1}{2} + \tfrac{1}{1} + \tfrac{1}{2} + \tfrac{1}{1} + \tfrac{1}{2} \text{ &c.}$$

summing five of these terms the fraction $\frac{19}{11}$ will be found, then

$$19^2 + 11^2 = 482$$
$$19^2 - 11^2 = 240$$
$$2 \times 19 \times 11 = 418$$

So that 241, 209, and 120, will form a rational right angled triangle, and two of these triangles placed together will form one nearly equilateral, the three sides will be respectively 240, 241, and 241, and the perpendicular will be 209. If eleven terms of the continued fraction be summed, $\frac{989}{571}$ is obtained,

$$\therefore \quad 989^2 - 571^2 = 652080$$
$$989 + 571^2 = 1304162$$
$$2 \times 989 \times 571 = 1129438.$$

These numbers may be divided by two, and then the three sides of the required triangle will be 652080, 652081, 652081 nearly equilateral, and the length of the perpendicular on 652080, will be exactly = 564719, a result easily verified.

*Ques.* 2. It is not possible by plane geometry to divide an angle into three equal angles, yet, the tangent, secant, and radius of one third of any plane angle, may be found, in whole numbers, as near as we please, in the following way. Let the angle to be trisected

$$= 41°.. 44' .. 33''.72.$$
$$3) \underline{41°.. 44'.. 33''.72}$$
$$13 .. 54 .. 51 . 24.$$

The natural tangent of this angle is found from the tables to be .2477386.

Let $x^2 - y^2 = .2477386$, and

$$2xy = 1.0000000$$

$$\therefore \quad x = .7993650, \; y = 6254964.$$

What follows is by continued fractions; the alternate quotients are marked, *first second*, &c.

| | | | |
|---|---|---|---|
| 7993650 | *first* | 6254964 | *second* |
| 6254964 | (1. | 5216058 | (3. |
| 1738686 | *third* | 1038906 | *fourth* |
| 1038906 | (1. | 699780 | (1. |
| 699780 | *fifth* | 339126 | |
| 678252 | (2. | 21528 | *sixth* |
| 21528 | *seventh* | 123846 | (15. |
| 16206 | (1. | 107640 | |
| 5322 | | 16206 | *eighth* |
| | | 15966 | (3. |
| | | 240 | |

$$\frac{377}{295} = 1 + \tfrac{1}{3} + \tfrac{1}{1} + \tfrac{1}{2} + \tfrac{1}{15} + \tfrac{1}{1} \text{ &c.}$$

$$377^2 + 295^2 = 229154$$
$$377^2 - 295^2 = 55104$$
$$2 \times 295 \times 377 = 222430$$

The three whole numbers 229154, 222430, 55104, forms a right angled triangle, one of whose angles will not differ from the third part of 41°.. 44'.. 33".72 by the third part of a second.

*Ques.* 3. Find a body as near the form of a cube as we please, whose edges, diagonal of the base, and of two opposite corners, may be expressed in whole numbers?

Let $x^2 + y^2 = (3)^{\frac{1}{2}}$, and

$$2xy = (2)^{\frac{1}{2}}$$

$$\therefore\ x + y = \left\{(3)^{\frac{1}{2}} + (2)^{\frac{1}{2}}\right\}^{\frac{1}{2}} = 1.7737712$$

$$x - y = \left\{(3)^{\frac{1}{2}} - (2)^{\frac{1}{2}}\right\}^{\frac{1}{2}} = .5637705$$

$$\therefore\ x = 1.1687708$$

$$y = .6050003.$$

To find the continued fraction;

| | | | |
|---|---|---|---|
| 11687708 | | 6050003 | |
| 6050003 | (1. *first* | 5637705 | (1. *second* |
| 5637705 | | 412298 | |
| 412298 | (13. *third* | 277831 | (1. *fourth* |
| 1514725 | | 134467 | |
| 1236894 | | 8897 | |
| 277831 | | 45497 | (15. *sixth* |
| 268934 | (2. *fifth* | 44485 | |
| 8897 | | 1012 | |
| 8096 | (8. *seventh* | 801 | (1. *eighth* |
| 801 | | 211 | |

$$1 + \tfrac{1}{1} + \tfrac{1}{1\,5} + \tfrac{1}{1} + \tfrac{1}{2} + \tfrac{1}{1\,5} + \text{\&c., \&c., \&c.}$$

this continued fraction summed gives for eight terms $\frac{1\,1\,8}{6\,1\,1\,8}\frac{1\,9}{}$.

$$11819^2 + 6119^2 = 177118685, \text{ (A)};$$
$$11819^2 - 6119^2 = 102258837, \text{ (B)};$$
$$2 \times 11819 \times 6119 = 144617284, \text{ (C)};$$

(B) is the length of each of the edges, (A) the exact length of the diagonal between two of the opposite corners. Further, if a four sided figure be constructed, each of its sides = (B), and (C), one of its diagonals, any one of the

four angles of this figure will not differ from a right angle two seconds. All the lines are in whole numbers without decimals. If only five terms of the continued fraction be summed, the result $= \frac{85}{44}$, then

$$85^2 + 44^2 = 9161$$
$$85^2 - 44^2 = 5289$$
$$2 \times 44 \times 85 = 7480$$

There are four exactly equal square faces, the length of each side = 5289, the distance between two of the most remote corners of this approximate cube = 9161 ; the four sided ends are equal, the side of each end = 5289, and one of its diagonals = 7480. None of the angles of the ends differ $\frac{1}{4}$ of a minute from a right angle.

The McCallum Inflexible Arched Truss Bridge, approaches nearer the standard of perfection than any other wooden bridge that have fallen under my notice, this fact is easily established by comparison and demonstration.

Fig. 1

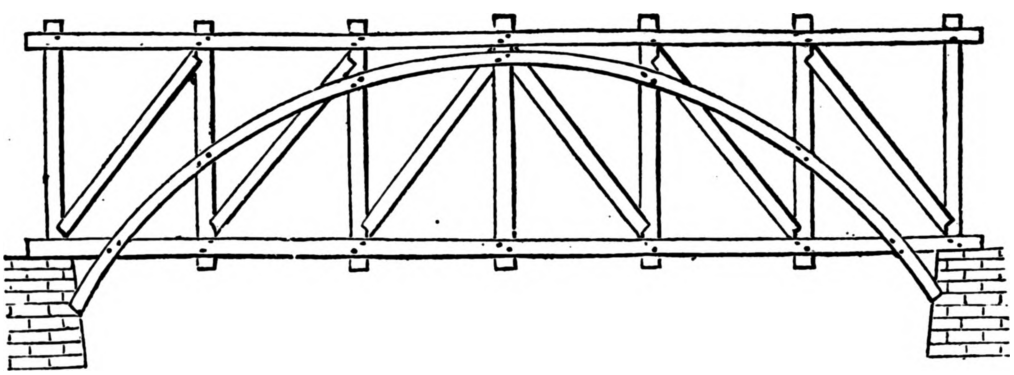


Fig. 1, is what is known as the *Burr Bridge.* It is composed of lower and upper chords, and posts and braces. The posts are framed into the chords, and the braces are framed into the posts. Arches are placed on each side of the truss, securely fastened thereto, and extending below the lower chords, abut against the masonry.

This form of truss was extensively used throughout the United States previous to the introduction of railroads. Many spans were of great length, and in cases where the arches were large, and the masonry sufficiently permanent,

this bridge was comparatively successful. Much difficulty was, however, experienced, by reason of the absence of counter braces. A moving load produced a vibratory and undulating motion, tending to loosen the connection of the timbers, which generally resulted in failure.

Many of the *first* railroad bridges in this country were built upon this plan, but much greater difficulty was found in adapting it to the use of railroads, than had been previously experienced in its use upon common roads. This difficulty arose from, 1st, the practical impossibility of perfectly combining the action of the *arch* and the *truss* (each system, of itself, being insufficient to carry the whole load); and 2d, the absence of counter braces. These defects, clearly apparent in their use on common roads, were greatly aggravated under the increased and concentrated nature of the weight, and the rapid transit of trains on railroads. It is true, they were obviated in part by adding largely to the amount of material in the structures; but, as the difficulty was inherent in the plan, violent contortions in shape could not be prevented, and these in time caused failures.

These remarks are intended to apply to spans of considerable length, as experience has proved that plans of even an inferior grade may be measurably successful in spans of ordinary length; whereas, nothing short of the most judicious distribution of material will insure permanency, in cases where long spans are indispensable, and any arrangement which can be made permanent in the latter case, must certainly prove so in the former.

It is worthy of remark here, that this particular combination of the arch with the truss, is even now with some, a favorite idea, but it is believed that its warmest advocates will be generally found among those whose opportunities for practical investigation have been limited, and that it is only necessary that the question be properly presented to them, to produce a change of views in respect to it.

This partiality for the combination of the arch and the truss is attributable partly to the fact, that the simple truss has in many instances failed, and as a last resort, the arch has been added, of such dimensions and strength, as to be competent to carry the truss and load, the truss serving only as a stiffener to the arch, while the latter, thrusting

upon the masonry, has sustained the whole weight. Besides, to the casual observer, who has never studied bridge construction, this combination presents at least *an appearance* of great strength and solidity, which do not in fact exist.

That the simple truss without the arch has failed in some instances, is unquestionably true ; but while many of these failures have been caused from inattention to, or ignorance of, the laws regulating the composition and resolution of forces, by far the greater nnmber have arisen from the inferior quality, or lack of the requisite amount of material, or from inferior workmanship.

The acknowledged failure of the *Burr Truss*, as applied to railroad purposes, led to the invention of several other plans, all of which were based upon the abandonment of the arch, and were aimed at perfecting a truss, which, *of itself*, would be sufficient to meet the emergencies of the case. This was in pursuance of what was considered a very resonable hypothesis, namely, that *one* system properly proportioned, must prove much superior to any method or arrangement in which the attempt was made to combine *two* distinct principles, in their nature heterogeneous.

Among the most prominent plans presented to remedy existing defects, was one invented by Col. Stephen H. Long. This plan of bridge was composed of lower and upper chords, posts and braces, similar in outline and general arrangement to the Burr Truss, but differing from it in detail. An efficient system of counter braces was introduced ; these were made adjustable by wooden wedges, as were also the sustaining braces, by means of which any desirable elevation or deflection might be given to the truss. This plan of truss was rigid to a degree not previously attained ; and to such an extent was this true, that when properly adjusted, no perceptible deflection was produced by the passage of the load.

It was, however, found difficult to keep it in adjustment, in consequence of the great shrinkage of the wedges and other timbers of the truss.

The invention of what is known as the *Howe Bridge*, followed. In this, as in Col. Long's bridge, the idea of combining the arch with the truss was *originally* abandoned, for reasons heretofore given, and it was believed that this

simple form of truss would prove equal to any reasonable requirement.

In the *Howe Bridge*, the posts used in the Burr and Long bridges are dispensed with, and iron rods substituted, by means of which any desirable *camber* may be given to the truss, thus overcoming the practical difficulty previously experienced in the adjustment of Col. Long's bridge, by the use of wooden wedges.

This method of producing *camber* is certainly an improvement upon the means adopted in the Long bridge, for that purpose, but is much inferior to the latter in its method of counter bracing, in that they are not adjustable, and perform a *negative* rather than a *positive* duty.

The *Howe Bridge* is composed of lower and upper chords, braces and counter braces, vertical rods, and cast iron *bearing blocks*. The braces abut the *bearing blocks*, which pass through the chords in such a manner as to permit the rods to bear directly upon them.

Spans of considerable length were built upon this plan, but experience proved that even this truss—like all others—had its limit, beyond which it could not be safely extended.

In the progress of railroad enterprises, in order to save large expenditures of money for masonry, longer spans than had been previously used became desirable, and in certain locations absolutely indispensable; besides this, locomotives were largely increased in weight, to meet the demands of traffic, and furnish a more economical mode of working, and thus arose the necessity for the adoption of some other expedient to meet the increased requirements of bridges. As all had been done by way of improving this truss that mechanical skill could devise. and which an extensive practice had amply afforded, it became evident, that some *radical* change must be made in its arrangment, to enable it to meet the exigencies of the case.

In this emergency the *arch*, heretofore condemned in the *Burr Truss*, was again resorted to, for it had been proved from the experience which its use in that truss had afforded, that an *arch* of sufficient size abutting against permanent masonry, would place the truss in a position of secondary importance.

It will be observed that the arch of the Burr Bridge,

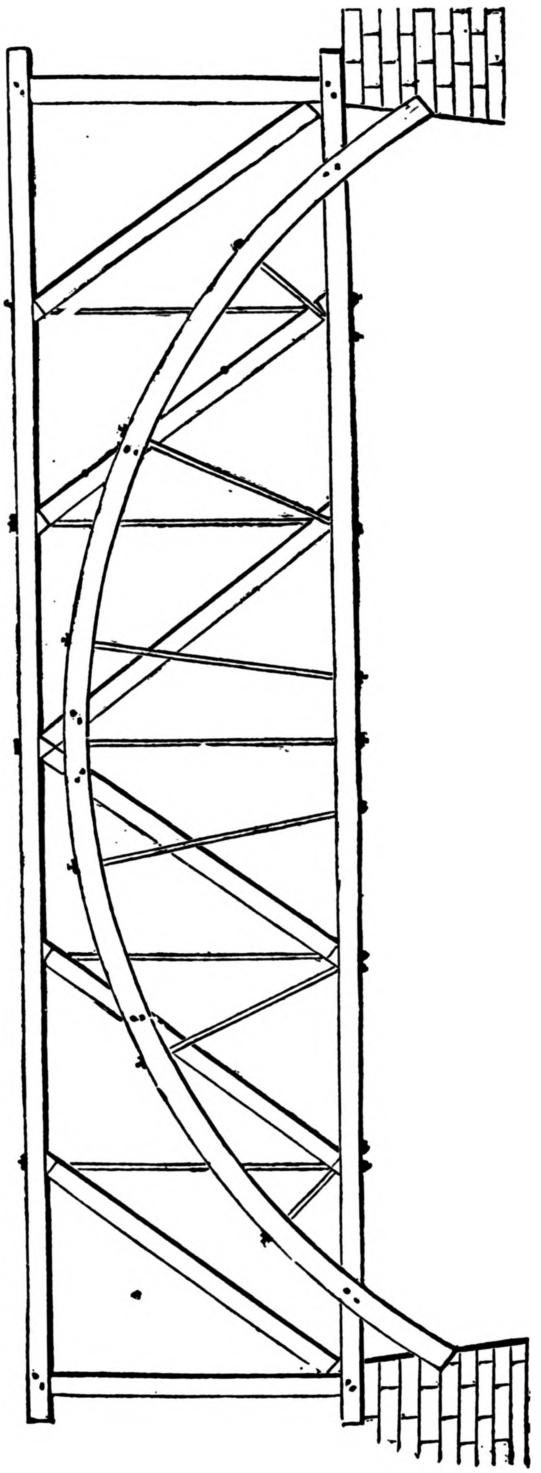


Fig. 2.

Fig. 1, abuts upon the masonry in precisely the same manner as the arch of what is denominated the *Improved Howe Truss*, Fig. 2, and the difference between the two consists simply in the mode of connection with the truss, and not in any change of principle, or method of action.

It will be seen that the Burr arch is securely fastened to the posts and braces of the truss, forming a solid adjustable mass. In Fig. 2, the arches are not fastened to the braces or rods, but have an independent connection with the lower chord of the truss, by means of rods radiating from the former to the latter. By this method it was supposed that any desirable adjustment could be effected, and that the strain could be put upon either system, or equally upon each.

This new arrangement, although plausible in theory, is found impossible in practice, for the following reasons:

1st. The rods from the arch to the lower chord are of various lengths, consequently their contractions and expansions must vary proportionately.

2d. Not a single rod in the arch is of the same length as those in the truss, hence the expansion and contraction of the rods in the truss will vary from that in each and all the rods connecting the arch with the lower chord.

3d. This combination is exceedingly liable to maltreatment, from the careless or ignorant.

4th. And even if it were everything in practice that is claimed for it in theory, (which is not the fact), it involves a constant expenditure for adjustment, which must continue during the existence of the bridge itself.

The Burr Truss, Fig. 1, with all its defects, *can be made superior by far* to the *Improved Howe Truss*, Fig. 2. For in the former, there may sometimes be a yielding and compression between the parts of the truss and those of the arch, producing a certain degree of united action; while in the Howe Truss, everything depends upon the length of the rods, which must always change with the temperature, and thus render an approach even to perfect adjustment, a matter of extreme delicacy.

But in either Fig. 1 or Fig. 2, it is clearly evident that, in order to have a structure absolutely safe, the arch and the truss—each of itself, independently of the other—should be of sufficient strength to sustain the whole load, that the strain may be borne alternately by each separate system.

In order to simplify and make clear the real points of difference existing in the combinations of the various plans of trusses, of the same general outline, it may be stated that the material composing *any bridge truss*, whether of wood or iron, or of both, is subjected either to *tension or thrust*, and it is upon the proper application of these elements, together with a judicious distribution of the material, rather than upon any difference in detail, that the perfection of any bridge structure depends; this may be illustrated by reference to Figures 3, 4, 5.

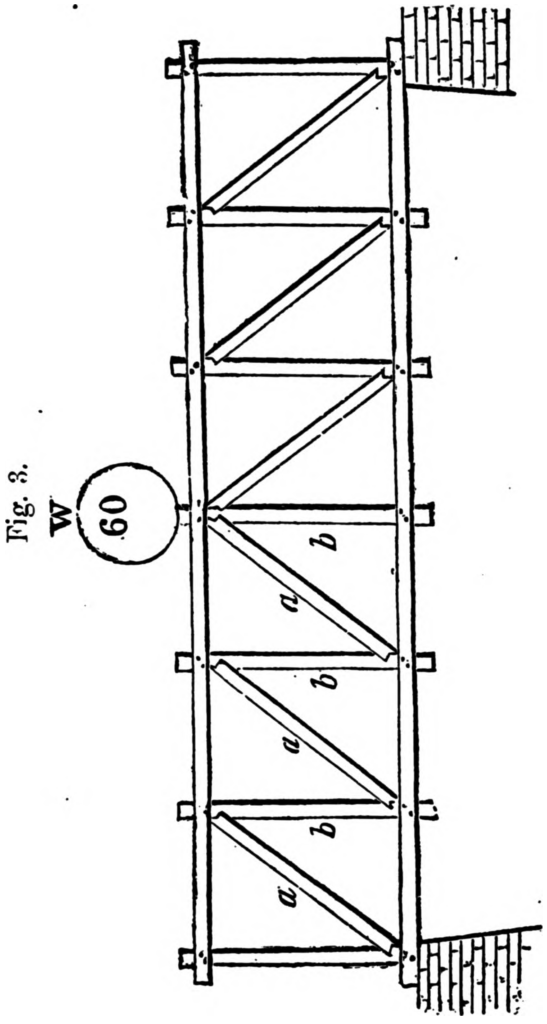



Fig. 3.

Figure 3 is the truss of the Burr Bridge; in this the upper chord and braces are acted upon by thrust, and the lower chord and posts by tension.

Figure 4 is the Howe Truss, without the counter braces;

in this also, the upper chord and braces are subjected to thrust, and the lower chord and vertical rods are acted upon by tension.

Fig. 4.

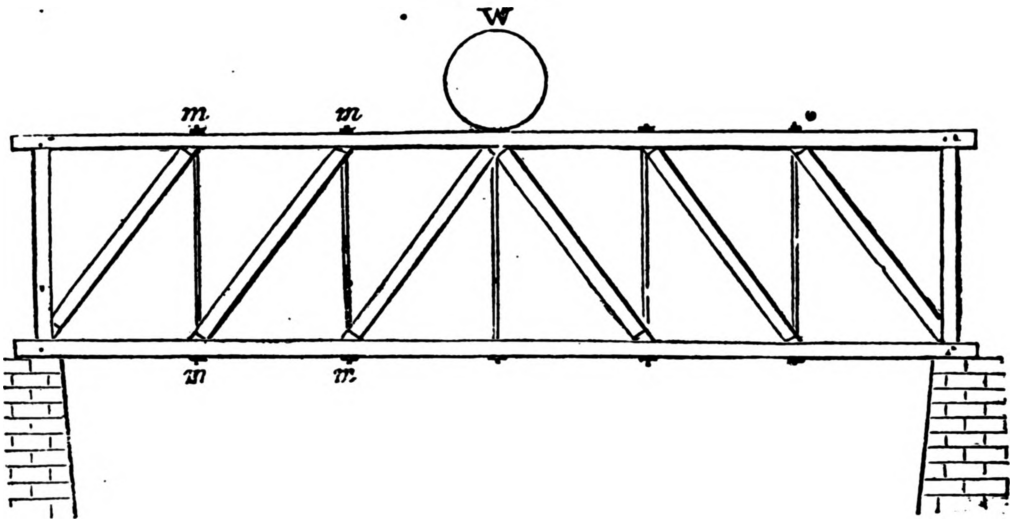
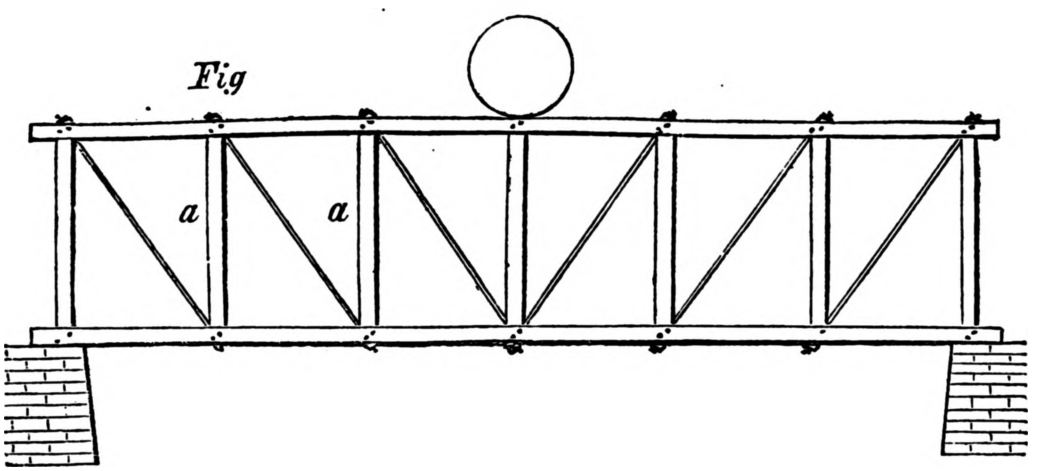



Figure 5 is a plan of truss sometimes used, the counter rods being omitted ; in this the upper chord and vertical struts are subjected to thrusts, and the lower chords and diagonal rods are acted upon by tension.

Fig. 5.




Upon a comparison of these plans it will be discovered that the variations between the Burr Truss, Fig. 3, and the Howe Truss, Fig. 4, consists in the use of vertical rods and bearing blocks in the latter, instead of vertical posts in the former, both having precisely the same duty to perform.

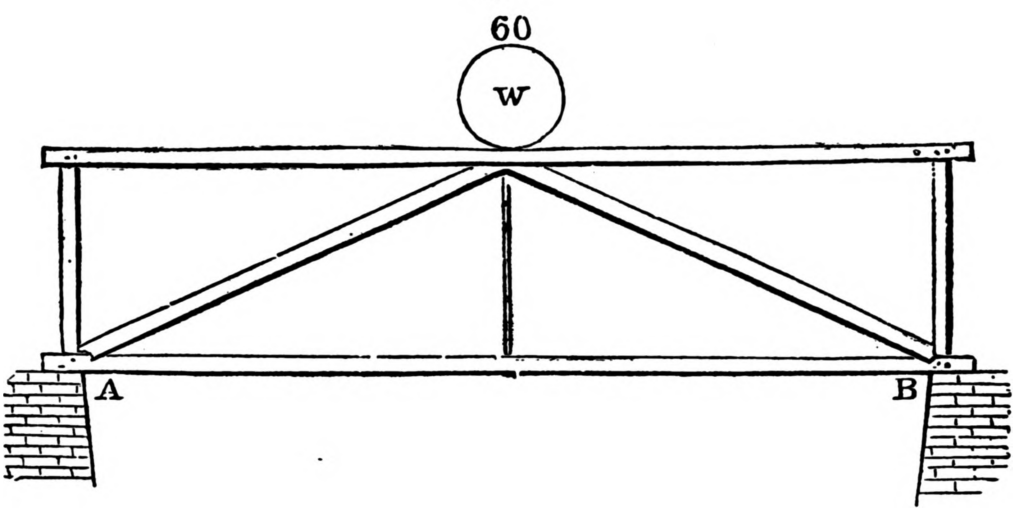
It will also be seen that Fig. 5 varies from Fig. 4, in that the rods are placed diagonally instead of vertically, changing the element of thrust from the diagonal braces in the latter, to the vertical struts in the former, and transferring the element of tension from the vertical to the diagonal line.

Much importance is sometimes attached to just such modifications in detail as exist in Figures 3, 4, 5, *while the nature and intensity of the destroying forces are the same and equal in each.*

This has been proved by actual experiment, by the celebrated engineer and bridge builder, D. C. McCallum, as follows: Models were built, one on each plan, of equal length and height of trusses, containing the same sectional area and kind of material in chords and braces, and of equal perfection in details and workmanship, when it was found that the real difference in strength was unappreciable, and it may be well to add, that any given amount placed upon each, in progress of the experiments, presented precisely the same characteristics and contortions in shape, until final failure took place.

All bridges having their chords parallel, irrespective of the particular method adopted in combining them, and re-

Fig. 6.




gardless of the amount of material used in their construction, when loaded to nearly the point of fracture, present somewhat the same appearance, the greatest deflection *being*

*invariably* at points near the abutments. This will be understood by the statement, that the vertical strain is increased, as the distance from the centre, to the ends of the truss ; at the centre the vertical strain is nothing, and at each end of the truss it is equal to one half the weight of the structure and its load.

In point of strength, the arrangements Figures 3, 4, 5, are not superior to the simple combination Fig. 6.

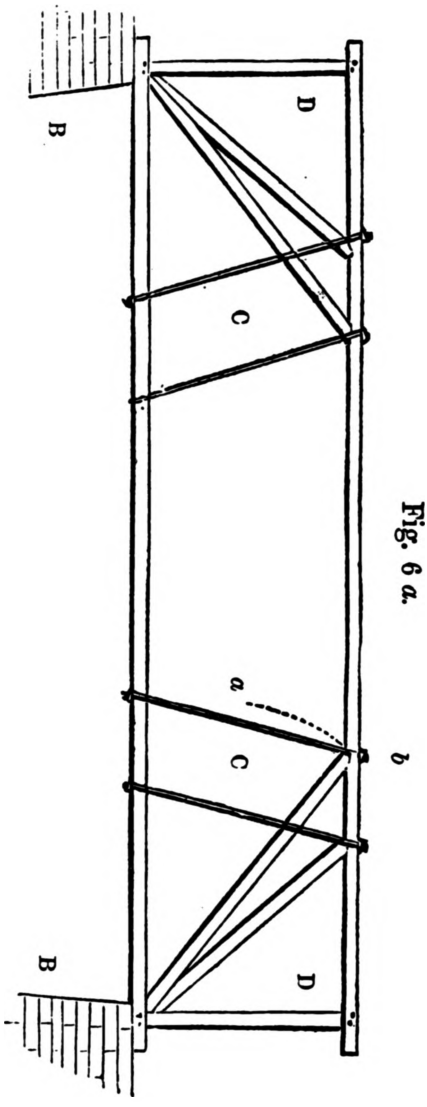



Fig. 6 *a*.

All bridges having their chords parallel, exhibit the same uniformity of action, and may be illustrated by reference to Fig. 6 *a*, in which A A, is the upper chord ; B B, the lower chord ; C, C, tension rods ; D, D, braces.

When a sufficient weight is applied to any truss of this outline, to cause deflection below the straight line, the upper ends of the braces D, D, are made to approach each other, and the distance between the ends is diminished, and as the deflection increases, the upper ends of the braces D, D, will describe arcs *a b*, of a circle *downwards*, the radius of which being the length of the braces D, D. But when

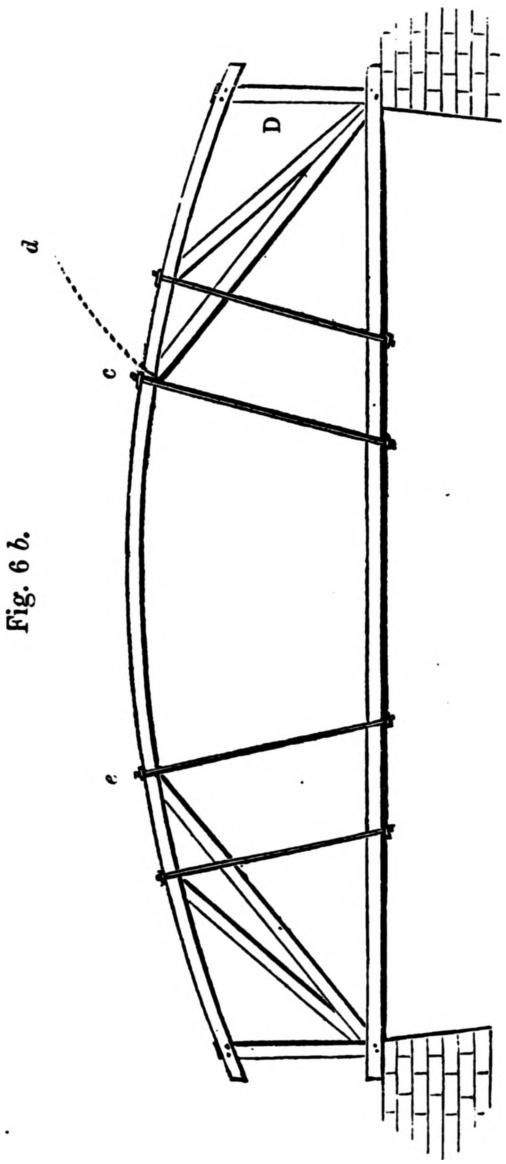



Fig. 6 *b*.

the upper chord is arched, as in Fig. 6 *b*, a sufficient weight will cause the braces D, D, to describe an arc *upwards*, represented by *c d*, Fig. 6 *b*. When the chord *c e* becomes

straight, the arc will then be described downwards, as shown in Fig. 6 *a*. As an illustration of the McCallum *Inflexible Arched Truss*, see Fig. 7, in which A, A, is lower chord ; B, B, upper chord ; C, C, tension rods ; D, D, braces ; E, E, struts, and W, weight.

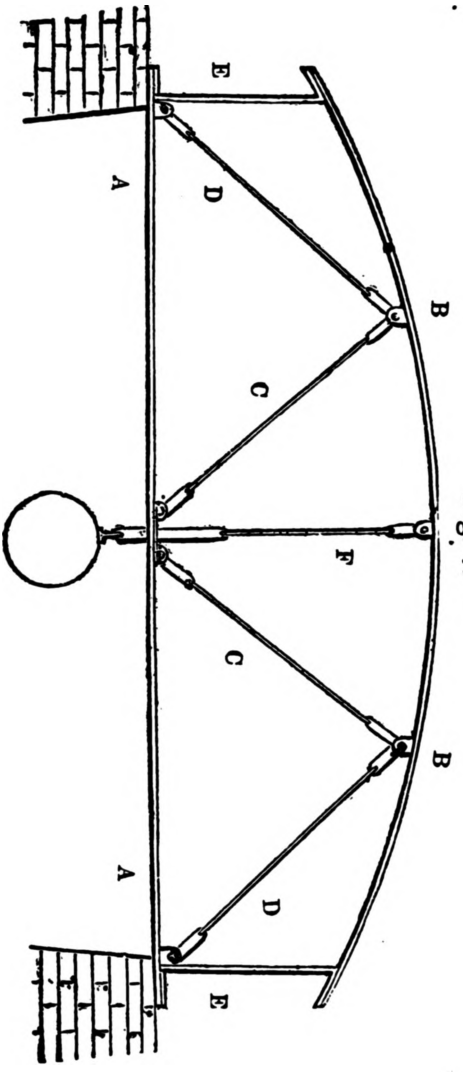



Fig. 7.

Upon an inspection of this figure, it will be seen that any deflection produced upon the centre of the arch, by means of the weight W, will cause the points B, B, to separate, by thrusting *outward*, and in the direction of the ends of the truss, producing an *upward* movement of the upper chord, at the ends of the braces D, D, the latter describing arcs of a circle *upward*, and from thence will be communi-

cated by means of the tension rods C, C, to the centre of the lower chord, raising the latter at the point where the rods C, C, meet.

By removing the weight W. and inserting a vertical strut at F, the upward movement of the chords will be arrested by the weight W. This peculiar action may be described as follows :

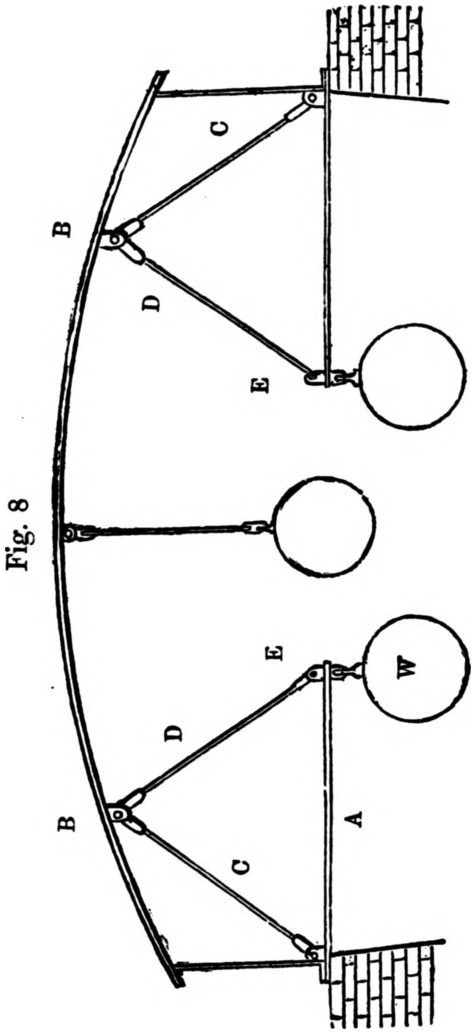



Fig. 8

Any deflection produced in the centre of the arch will cause an *outward*, and consequently, an *upward* force, at the upper ends of the braces, which, by means of the tension rods and strut, is transferred directly back to the under side of the arch, producing an *upward* force at the latter point, equal to the original *downward* force applied on top of the same.

This combination of forces is in agreement with a well-known law, namely, when two forces of equal powers of resistance are opposed to each other, a state of rest is produced.

For a further illustration of the action of this truss, see Fig. 8, in which A, A, are pieces of the lower chord, the centre being removed, B, B, upper chord, deflected by the weight W. C, C, are braces which pass through the lower chord, and rest upon the masonry. D, D, are tension rods. It will be seen that the ends of the pieces of lower chord at

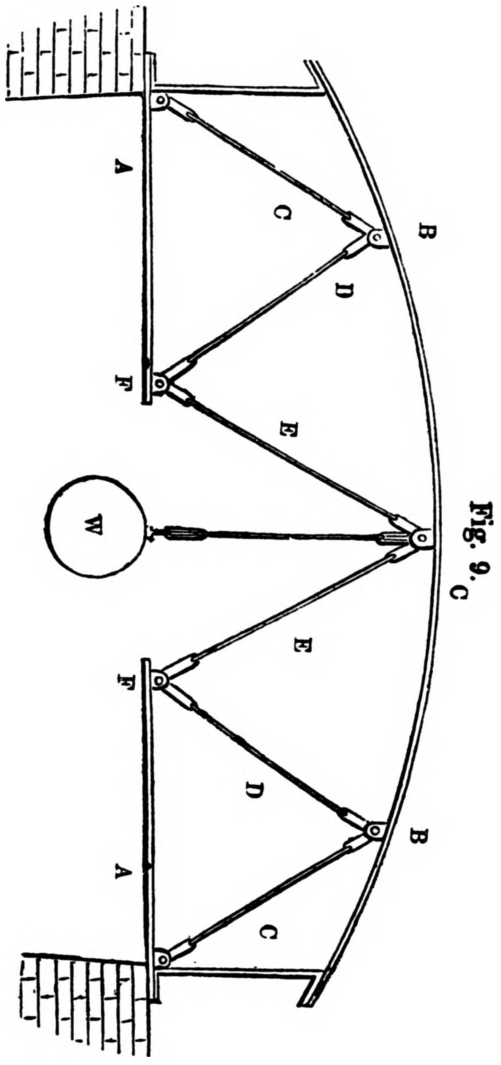



Fig. 9.

E, E, are raised considerably above a horizontal line. This upward tendency will continue until the upper chord between B, B, is deflected below a straight line, when the action will be reversed.

Figure 9 exhibits the forces at a state of rest, in which A, A, are portions of the lower chord; B, B, upper chord; C, C, arch braces, which pass through the lower chord, and rest in the masonry. D, D, tension rods; E, E, braces; W, weight.

It will be seen that the strain produced by the weight W, is transferred to the lower chord by means of thrust upon the braces E, E, to the points F, F, and by means of tension on the rods D, D, to the points B, B, and from thence it is brought upon the arch braces C, C, which rest upon the masonry.

In this manner, a perfect equilibrium of forces is effected, as it is evident that the point G, cannot change position, unless the points B, B, are thrust *outward* towards the ends of the truss, which must raise these points, this being prevented by the strain upon the points F, F, communicated by the weight W, through the braces E, E.

For a full plan of McCallum's Inflexible Arched Truss, the reader is referred to Fig. 10. Upon inspection, it will be observed that the sustaining principle is very much increased toward the ends of the truss, not only by the addition to the amount of material at these points, but it will be seen also that the pannels become shorter as the vertical strain increases. The posts are placed upon lines radiating with the arch; the braces form equal angles with the posts; and in this way the latter are made to approach more nearly together toward the ends of the truss.

The student has already had sufficient evidence of the great strength of this form of truss, and it has also been shown, that the tensile strain upon the lower chord is much less than in any other known plan. In fact, the latter may be *entirely severed*, and the structure will still be competent to sustain a heavy load. In this, it differs from all other combinations.

Upon referring to Figure 10, which represents a clear span of 180 feet, it will be seen that the arch braces which rest upon the abutments are extended to points on the arch about forty-seven feet from the abutments. From the top of each set of arch braces, running diagonally on each side of the truss, are placed heavy suspension rods, which are connected with the lower chords 12 feet further from the masonry. Thus the bridge seat is substantially transferred

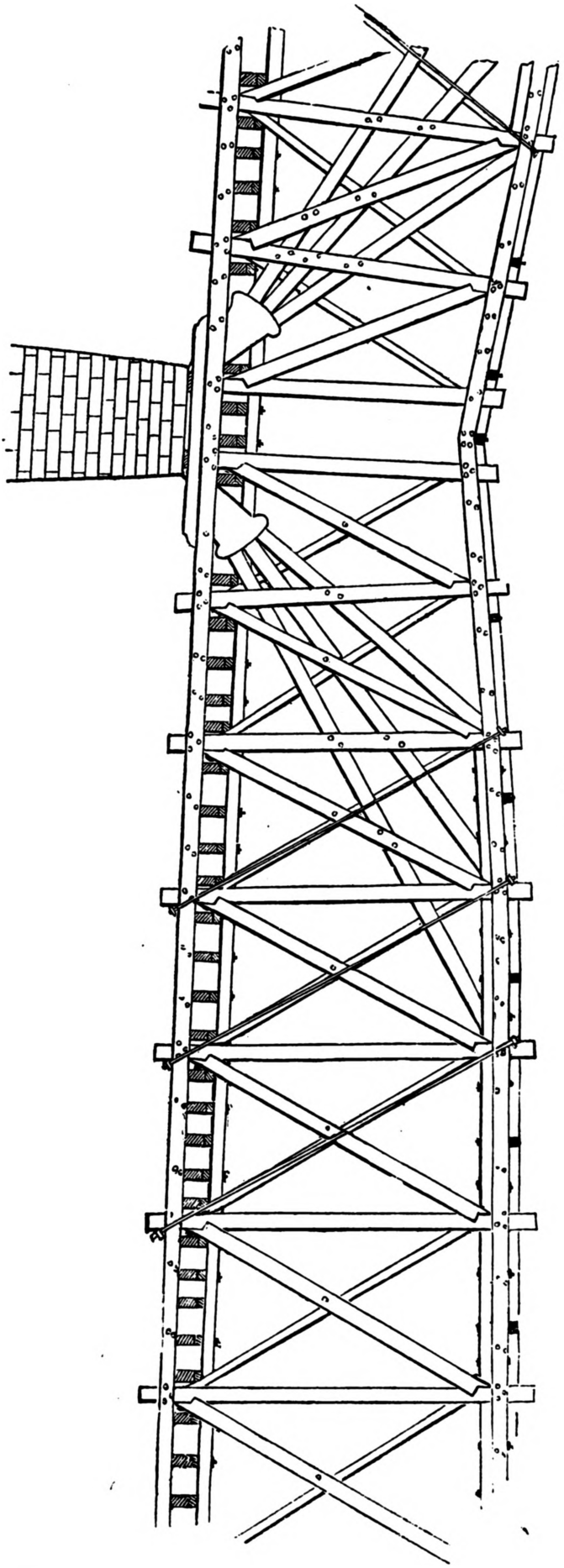


Fig. 10.

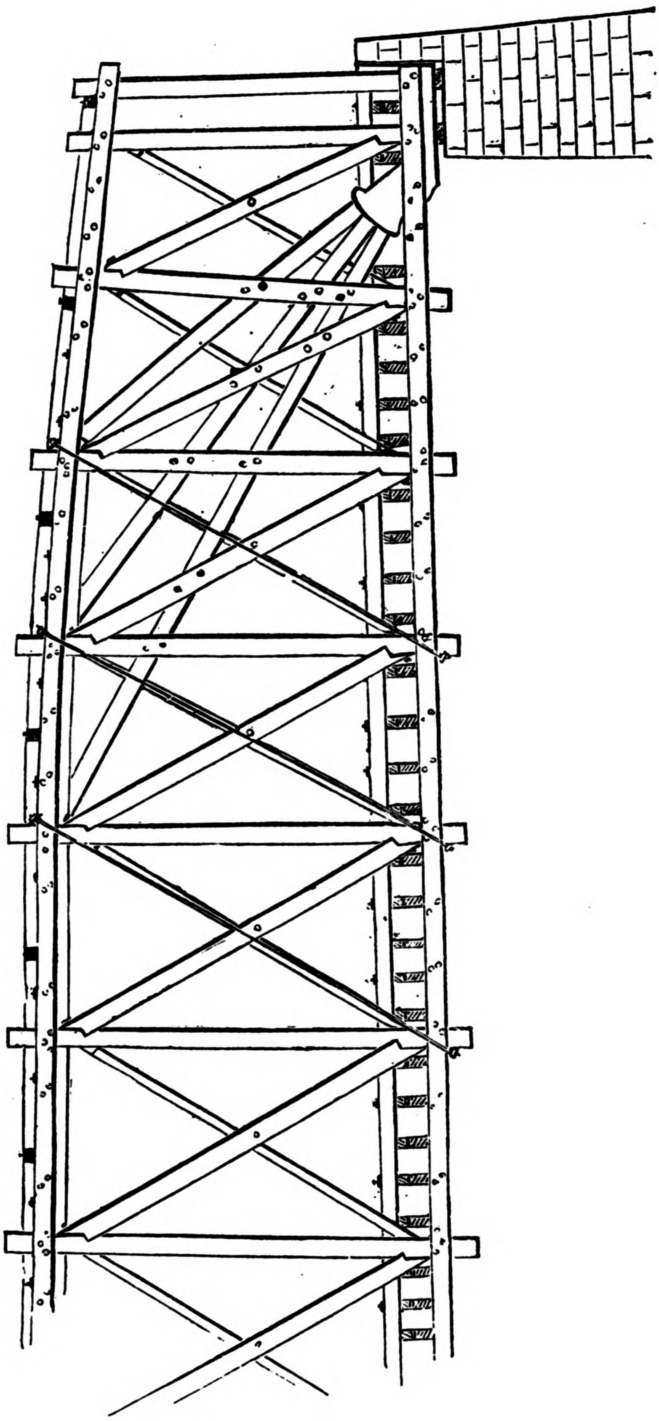


Fig. 10.

to a point 47 feet towards the centre of the bridge, reducing a span of 180 to 86 feet, so far as the *tensile strain* upon the lower chord is concerned.

For this intermediate space of 86 feet, the arch beam is of sufficient strength to sustain the whole load, if required.

*Strength*, however is not all that is required, for a Railroad bridge especially, subject as it is to a *moving* load, there must also be *rigidity*, *stiffness*, freedom from vibration. A bridge *may* be strong yet flexible, rigid yet weak ; in fact, flexibility is incompatible with durability, the structure should be prepared at all times to receive its load, and should not be permitted to change shape in the slighest degree, by its passage over it.

To produce this result, an effective system of counter braces, is indispensible.

The proper office of counter braces is frequently misunderstood, as is evident from the manner of their application in many cases in which they are used as *check braces only*, having a *negative* rather than a *positive* action ; this may be readily shown. When the load is applied, the truss is deflected in consequence of the yielding of the braces ; this has the effect of *shortening* the diagonals in the direction of their length, while the diagonals in the direction of the counter braces are correspondingly *lengthened ;* this, will leave a space between the ends of the latter, and the *bearing block* in the lower chord.

When the truss is in this condition, if wedges are inserted between the ends of the counter braces and the lower chord, in such a manner as to fill up the whole space, it is evident that the weight may be removed without at all affecting the shape of the truss, the deflection originally produced by the weight, being maintained by the counter braces, the strain upon the sustaining braces and other portions of the truss remaining precisely the same as when the weight was suspended.

Now suppose the original weight to have been 200 tons, it is evident that as soon as it is removed, each counter brace will be subjected to an upward thrust, easily found from its position ; the sum of all the thrusts making 200 tons.

Now let there be a smaller load applied, this load will not produce any additional strain upon any portion of the truss, nor will the deflection be increased in the slightest degree ; the only effect produced by suspending the latter weight, will be the relief of the counter braces, equal to the difference between the first and second weights.

The inventor has found it very difficult to explain this

clearly in the course of conversation with some individuals, from the fact that *weight* and *strain* were confounded. Now it is true, when the original weight was applied of 200 tons, the abutments were loaded with just 200 tons more than previously, and the truss was also loaded with ten tons more; but when the wedges were driven, and the weight removed, while the abutments were relieved of 200 tons pressure, the truss still retained the original strain produced, the weight being required to *produce the strain*, the latter remaining after the former has been removed.

In order to make a practical application of the above, the following method of adjusting the *Inflexible Arched Truss*, is submitted. When these bridges are raised, it is usual to load them with a train of locomotive engines, attached closely to each other, and that greater weight may be obtained, the tenders are sometimes detached, and the bridge covered with engines only; with this load, the latter is strained down to a perfect bearing in all its parts; by this means the whole structure is more or less deflected, while the counter braces are hanging loosely in their places; if, therefore, when the bridge is in this condition with its load, the counter braces could be lengthened with considerable force, it would not recover its original shape upon removal of the load, but would be held down by the action of the counter braces to very nearly the same position as when loaded. In this plan of bridge, the lower ends of the counter braces rest in *iron stirrups*, which are attached to the vertical ties or posts at a point near the lower chord by means of castings and nuts, by which they may be lengthened several inches, in this manner they are made to perform a *positive* duty. When the bridge is adjusted as above, it is clear that a *less* load than that originally applied cannot produce any deflection whatever, the only effect of the passage of a train over it will be to relieve the counter braces, and will not add a pound pressure upon any timber of the trusses.

In the arrangement of any bridge truss, the attainment of the following requisites is desirable:

*First*, Such equilibrium of forces as will produce uniformity of action.

*Second*, Such quantity and distribution of material, as will insure *a large surplus* of sustaining principle *thereby guarding the structure against accident.*

*Third*, *Perfect rigidity*, that the combination in all its parts may have permanency *equal to the durability of the material composing the same.*

*Fourth*, The arrangement of the parts should be such as to be free if possible, from *the necessity of adjustment.*

The McCallum *Inflexible Arched Truss* meets all these requirements.

PROBLEM.

LET it be required to find the equal weights $w$, $w$, Fig. 11. Kept in a state of rest by a single weight W, which has caused the arc $q\ C\ a$ to assume the chord $q\ D\ b$, the rigidity of the arc being neglected.

Let R = the radius of the arc $q$ C $a$, and put $2\,c$ = its length; then the chord $q\,a$ and versine C D are readily calculated.

Let $q\,a = 2\,d$ and C D $= 2\,e$.

Let $f =$ A $q =$ A $p =$ B $a =$ B $b$ and

$$g = \text{B}\,n = \text{B}\,m = \text{A}\,s = \text{A}\,r.$$

Also, let A B $= 2\,h$

$$\frac{2\,h - 2\,d}{2} = h - d,$$

$$\frac{h - d}{f} = cos.\ \theta,$$ putting $\theta$ for the angle $a$ B $n = q$ A $s$.

$$\frac{2\,h - 2\,c}{2} = h - c,$$

$$2\,h - 2\,c = \text{A B} - q\,b.$$

∴ Putting $\varphi$ for the angle $b$ B $n = q$ A $s$,

$$\frac{h - c}{f} = cos.\ \varphi.$$

∴ $\varphi - \theta =$ angle $a$ B $b = n$ B $m = s$ A $r$.

∴ $m\,n = g\ sin.\ (\varphi - \theta)$.

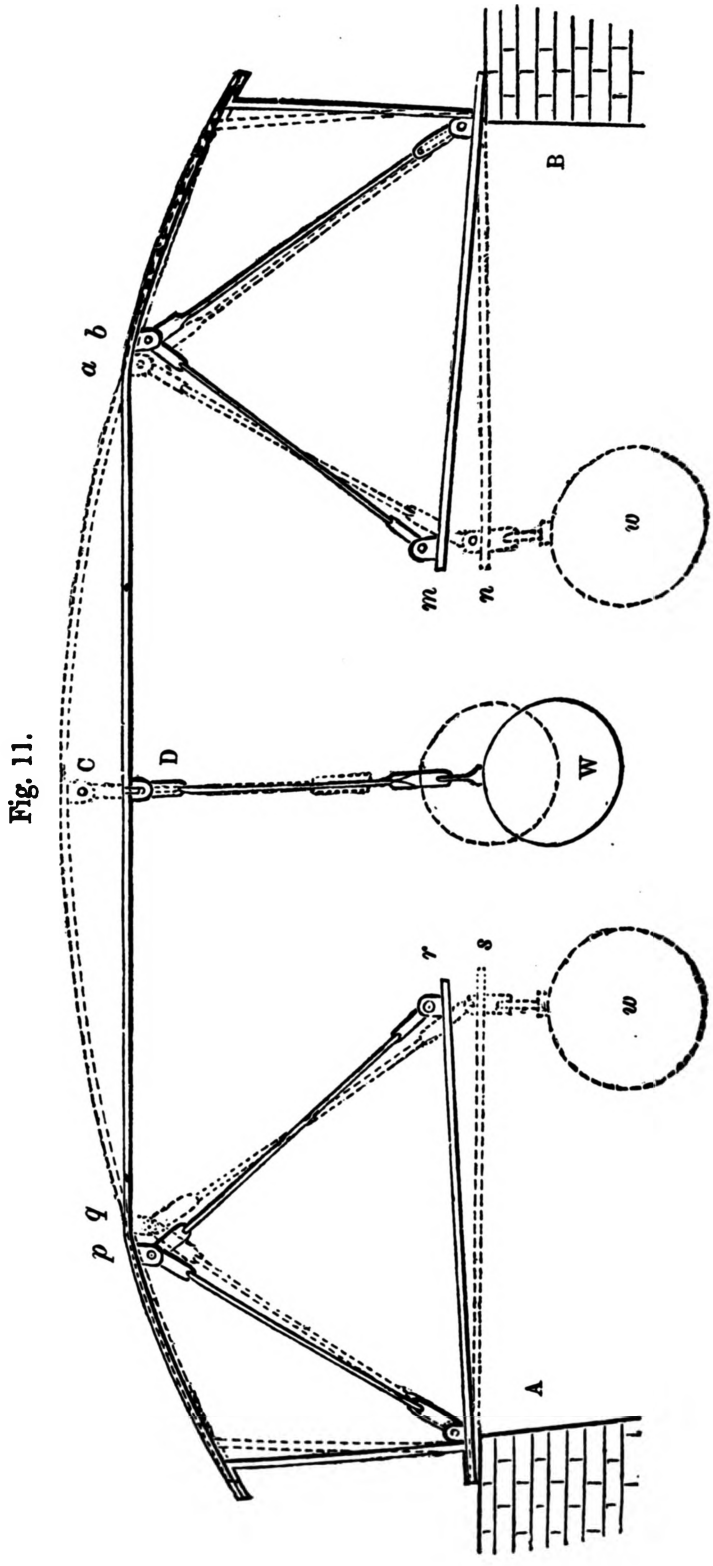



Fig. 11.

Then according to the principle of work,

$$2\,g\,sin.\,(\varphi - \theta)\,w = 2\,e\,W.$$

$$\therefore\quad w = \frac{e\,W}{g\,sin.\,(\varphi - \theta)}.$$

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## IRON SUSPENSION AND TRUSSED BRIDGE AT HARPER'S FERRY—DESIGNED BY WENDEL BOLLMAN.

THE span of the Iron Suspension and Trussed Bridge, at Harper's Ferry, is 124 feet between the abutments. The length of cast iron in the stretcher is 128 feet. The weight of cast iron is 65137 lbs.; weight of wrought iron 33527 lbs., making the total weight of cast and wrought iron 98664 lbs.

COHESIVE STRENGTH PER SQUARE INCH OF CROSS-SECTION.

| KIND OF MATERIALS. | JUST TEAR ASUNDER. | | WITH SAFETY. | |
|---|---|---|---|---|
| | *Pounds.* | *Tons.* | *Pounds.* | *Tons.* |
| Cast Steel,..... | 134256 | 59.93 | 33600 | 14.98 |
| Blistered Steel..... | 133152 | 59.43 | 33300 | 14.86 |
| Steel, Sheer,..... | 128632 | 56.97 | 32160 | 14.24 |
| Iron, Swedish bar,..... | 65000 | 29.2 | 16260 | 7.3 |
| " Russian,..... | 59470 | 26.7 | 14900 | 6.7 |
| " English,..... | 56000 | 25.0 | 14000 | 6.25 |
| " common, over 2 inches square | 36000 | 16.00 | 9000 | 4.0 |
| " sheet, parallel rolling,..... | 40000 | 17.85 | 10000 | 4.46 |
| " at right angles to roll,..... | 34400 | 15.35 | 8600 | 3.84 |
| Cast iron, good quality,..... | 45000 | 20.05 | 11250 | 5.00 |
| Platinum, wire..... | 53000 | 23.6 | 13250 | 5.9 |
| Brass, common,..... | 4500 | 20.05 | 1125 | 5.0 |
| *Wood.* | | | | |
| Ash..... | 16000 | 7.14 | 4000 | 1.87 |
| Beach,..... | 11500 | 5.13 | 2875 | 1.28 |
| Box,..... | 20000 | 8.93 | 5000 | 2.23 |
| Cedar,..... | 11400 | 5.09 | 2850 | 1.27 |
| Mahogany,..... | 21000 | 9.38 | 5250 | 2.34 |
| " Spanish,..... | 12000 | 5.36 | 3000 | 1.34 |
| Oak, American white,..... | 11500 | 5.13 | 2875 | 1.28 |
| " English,..... | 10000 | 4.46 | 2500 | 1.11 |
| Pine, Norway,..... | 13000 | 5.8 | 3250 | 1.45 |
| Walnut,..... | 7800 | 3.48 | 1950 | 0.87 |
| Hemp ropes, good,..... | 6400 | 2.86 | 2130 | 0.95 |
| Iron chain,..... | 65000 | 29 | 21600 | 9.38 |
| " with cross pieces,..... | 90000 | 40 | 30000 | 13.4 |

Fig. 12. is an elevation of part of the side, showing one pier, part of the cast iron stretcher. The cap is removed from the pier to show how the rots are secured.

Fig. 12.

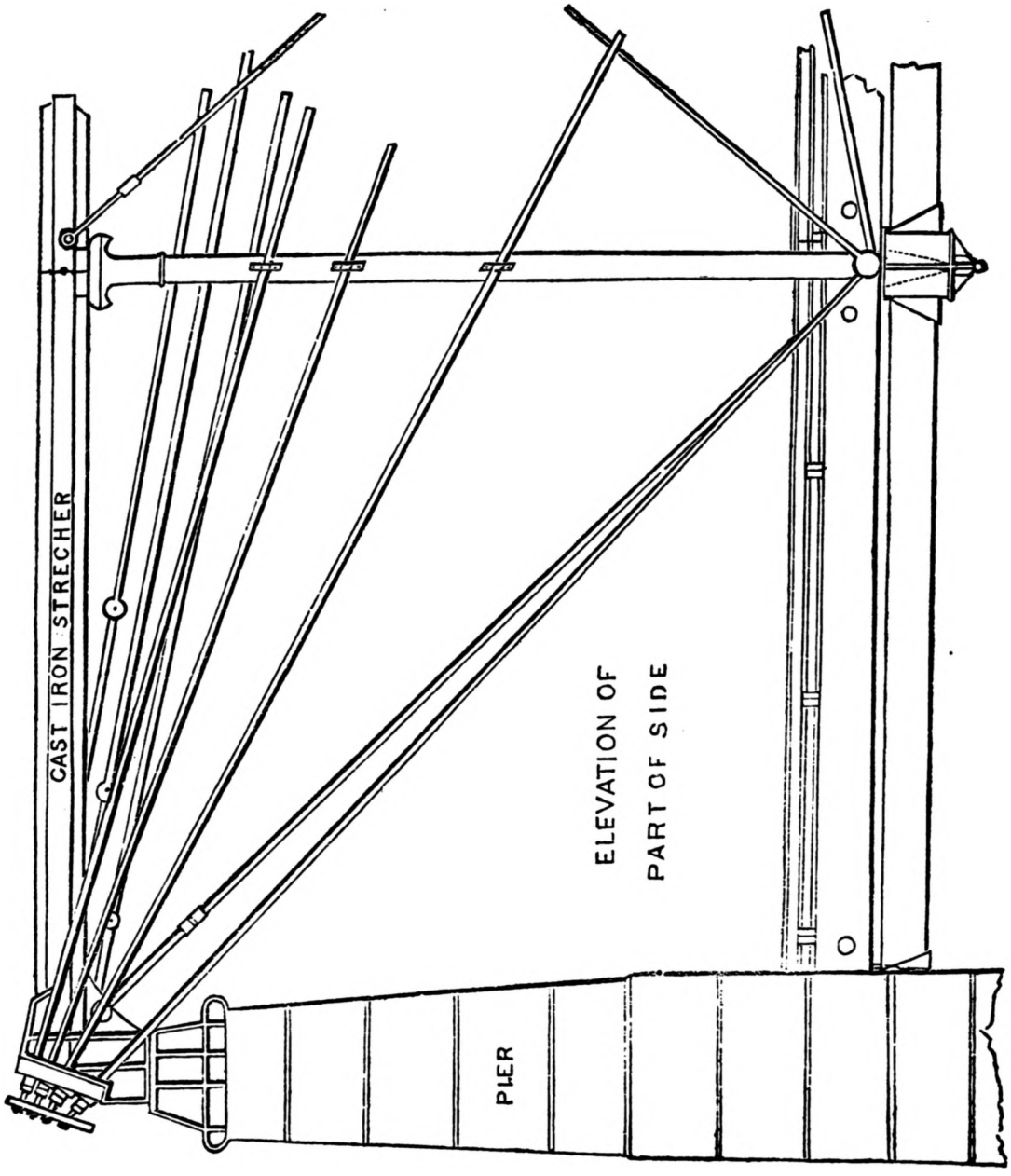



Fig. 13, is an elevation of a pier and four pannels out of eight of which the bridge is composed. The system of arranging the braces and connecting rods is exhibited in this figure.

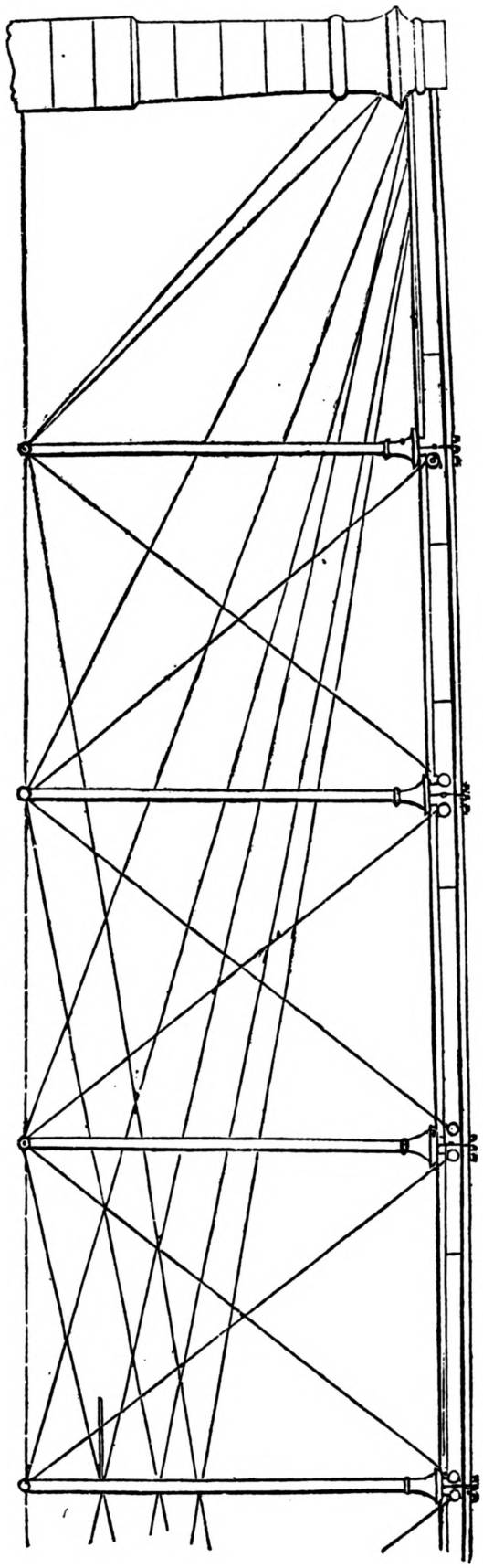


Fig. 13.

Fig. 14, is cross section, showing the floor bracing, and the position of the rails. Fig. 14, also shows in section, the roof and posts.

Fig. 14.

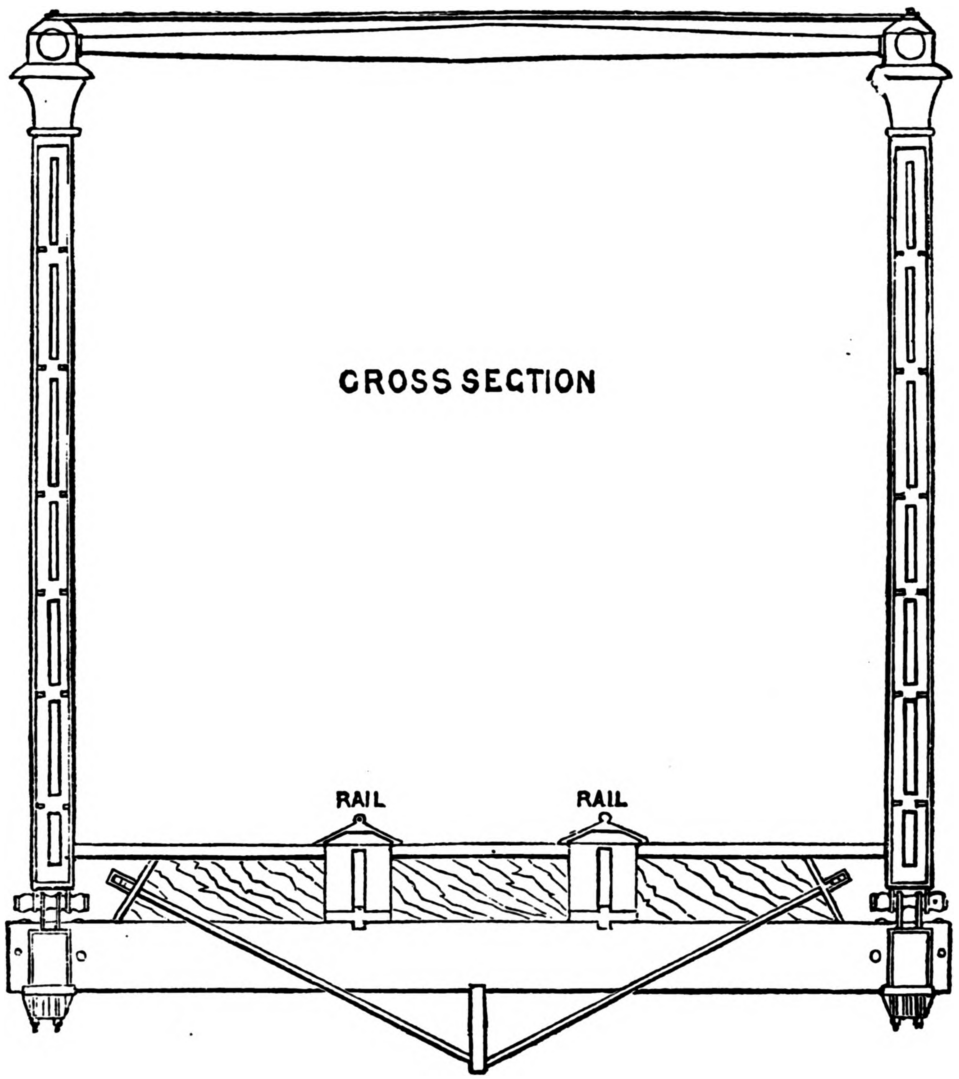



Fig. 15, shows a plan of the flooring of the bridge, the positions of the rails and floor bracing.

Fig. 16. shows two posts part of the stretcher, and the diagonal rods in one of the pannels.

The wrought iron requires little workmanship ; the rods from the centre to abutments having but an eye at one and a screw at the other, end ; with a weld or two between, according to length. The long counter rods have two

Fig. 15.

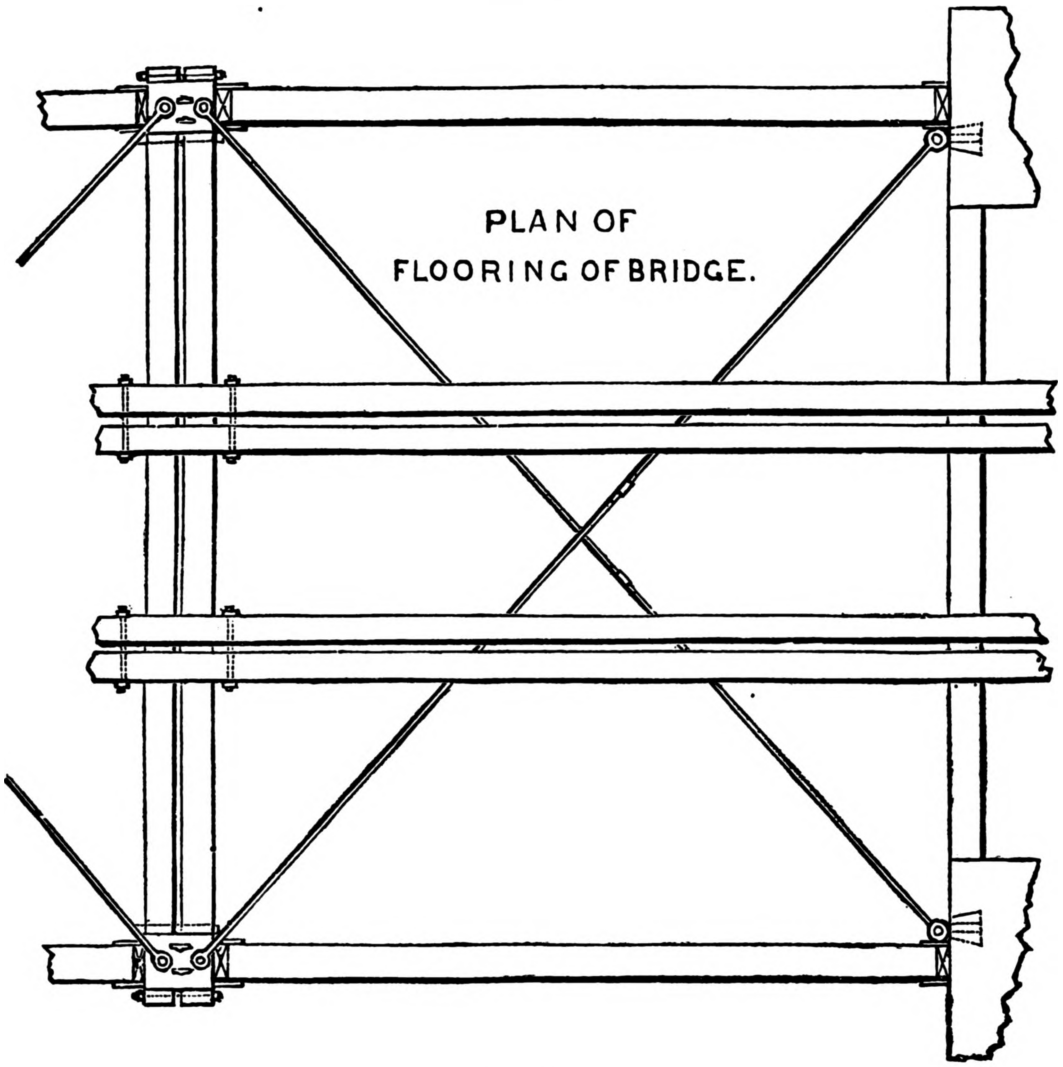
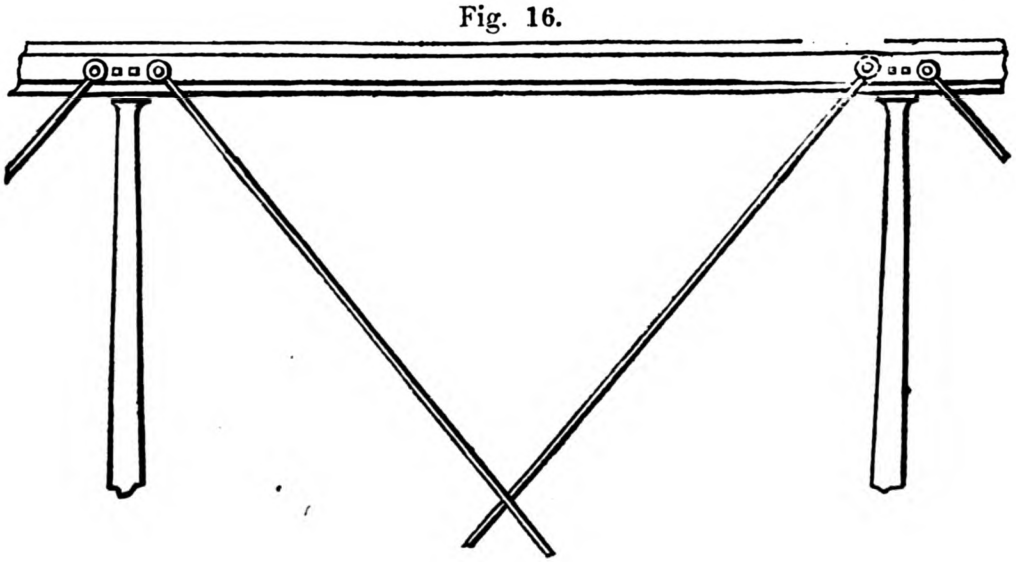



Fig. 16.



knuckles and one swivel for adjustment of strain, and convenience in welding, as well as in raising the whole.

The cast iron stretcher is octagonal without, circular within, and averages one inch of metal. It is cast in lengths according to the length of panel, and jointed in the simplest manner ;—at one end of each length is a tenon, at the other a socket. The latter is bored out, and the tenon and its shoulder turned off in a lathe to fit the socket ; thus, when thoroughly joined, to form one continuous pipe between abutments. The ends of the sections of cylinders, inserted to those contiguous, are slightly rounded, to allow a small angular movement without risk of joint fracture.

A cast iron plate or washer, sets on a bracket cast with each abutment end of stretcher, and at right angles to the centre acting rods. The tension bars are passed through this washer to receive a screw nut for the erection and adjustment of the system.

The stretcher or straining beam, the vertical posts, and suspension bars compose the essential features of the bridge ; each post being hung by two bars from both ends of the stretcher independently of all the others ; and each post and pair of tension bars forming with the stretcher a separate truss.

This system, perfect in itself, is additionally connected by diagonal rods in each panel ; also by light hollow castings, acting as struts. The diagonal side rods might be safely dispensed with, for the peculiar merit of the truss is its perfect independence of such provision. They are therefore used as a safeguard only in case of the fracture of any of the principal suspension rods.

By this combination of cast and wrought iron, the former is in a state of compression, the latter in that of tension ; the proper condition of the two metals. It unites the principles of the Suspension and of the Truss Bridges. *Each* bar performs its own part in supporting the load in proportion to its distance from the abutment ; so that the entire series of suspending rods transmits the same tension to the points of support as would be equally transmitted from thence to the centre of the bridge.

This bridge. it will be seen, is composed of seven independent trusses, which transfer the weight concentrated on

each floor-beam directly to the abutments, without aid from any other connection ; and not from panel to panel as in general use.

The strain on cast and wrought iron is wholly in direct line ; and the result, the least quantity of metal is required to carry a given weight. The weight of bridge and load has a vertical pressure on the piers, towers, &c., the only horizontal thrust being from the expansion of iron, which is accommodated by rollers, sliding of abutment bracket over its pedestal, or by other means : the necessary dimensions of masonry may therefore be most moderate.

It is evident, from an inspection of the drawing, that no chord is requisite at the bottom of the truss to resist tension ; the only advantage of that employed is to regulate the movement produced by expansion, in the performance of which agency the resistance is one to compression.

Although the abutment bracket casting and its pedestal were so constructed as to admit of accommodation to expansion, by rollers, yet such contrivance was omitted with the view of fully testing the effect of greatest expansion throughout the system.

It is now ten months since this bridge was erected at Harper's Ferry ; during which time it has been exposed to extremes of cold and heat, and to an average run of twenty trains daily.

From the closest inspection, we find that the extreme expansion measures, as near as possible, five-sixteenths of an inch on each tower, or five-eighths in the entire length, 128 feet of stretcher ; and without the slightest perceptible derangement of masonry ; the dimensions of which are 4 feet square of base, 12 feet high, and 2 feet 9 inches at top.

While on the subject of expansion, it may be well to notice the effect from difference in expansion of the rods. At the first point of suspension, or where the longest and shortest rods meet, the counter rod is about four and a half times longer than the acting rod ; and the expansion of the counter is four and a half times that of the acting rod. But there is also a proportionate difference in the lengths of stretcher from the point directly over the centre of connection to the extremities of these rods. This has been practically proved in this bridge.

The suspender bolt, when the expansion is extreme or five-eighths of an inch in length of stretcher, exhibits a motive difference of three-sixteenths toward the short or acting rod ; which difference is provided for, as seen by slot-dotted in Elevation, where the vertical suspender bolt moves to accommodate any such difference, and to give that proportion of weight to each rod according to the angle.

It affords easy access for repairs ; for instance, should a new floor beam be required, it is but needed to slacken the horizontal rod and the keys in longitudinal strut, remove the washer under point of suspension, and let down the beam to be replaced ; which can be done without trestling up any part of the bridge.

In case of fire, the floor may be entirely consumed without any injury to the side truss.

The permanent principle in bridge building, sustained throughout this mode of structure, and in which there is such gain in competition with every other, viz : the direct transfer of weight to the abutments, renders the calculation simple, the expense certain, and facilitates the erection of secure, economical, and durable structures.

### *Trial made on the* 1*st day of June* 1852, *to prove the capability of this Bridge.*

Three first-class tonnage engines, with three tenders, were first carefully weighed, and then run upon the bridge, at the same time nearly covering its whole length, and weighing in the aggregate 273,550 lbs., or $136\frac{1}{2}\frac{5}{0}\frac{5}{0}\frac{0}{0}$ tons nett, being over a ton for each foot in length of the bridge.

This burden was tried at about eight miles per hour, and the deflections, according to guages properly set and reliable in their action, were at centre post, $1\frac{3}{8}''$, and at the first post from abutment nine-sixteenths of an inch.

From this test it is found, that the load did not cover the entire length of bridge by about 13 feet, yet the excess of weight in the middle, and at a speed of about eight miles per hour, produced no greater deflection than $1\frac{3}{8}$ of an inch at the centre post, and nine-sixteenths of an inch at the first point from abutment.

## EXAMPLES FOR PRACTICE.

### PROBLEM I.

To find the difference between the logarithms of two consecutive numbers without knowing the logarithms of the numbers themselves, and to calculate the logarithm of any given number.

#### RULE.

*When the numbers are greater than* 100, *divide* .868589 *by double the lesser number plus one, and the quotient will be the difference of the logarithms of the given numbers.*

#### EXAMPLES.

*Question* 1. What is the difference between the logarithms of 125 and 126, without finding the logarithms of either?

$$2 \times 125 + 1 = 521.$$

$$\frac{.868589}{251} = .003461$$

So that the difference of the logarithms of 125 and 126 is = .003460.

*Ques.* 2. What is the difference between the logarithms of 126 and 127?

$$\frac{.868589}{253} = .003433$$

*Ques.* 3. What is the difference between the logarithms of 127 and 128?

$$\frac{.868589}{255} = .003406.$$

Hence, the difference between the logarithms of 125 and 128 becomes known, without knowing the logarithms of either 125 or 128, or of the intermediate numbers.

.003406
.003433
.003461

.010300

The difference between the logarithms of the numbers 125 and 128, is found to be .010300, which I will put $= n$ in the next example.

*Ques.* 4. Required the logarithms of the numbers 2 and 5 ?

$$5^3 = \left(\frac{10}{2}\right)^3 = 125$$

$$2^7 = \quad = 128$$

$$log.\ 128 - log.\ 125 = n = .010300$$

$$\therefore\ log.\ (2)^7 - log.\left(\frac{10}{2}\right)^3 = n$$

$$\therefore\ 7\ log.\ 2 - 3\ log.\ \frac{10}{2} = n$$

$$\therefore\ 7\ log.\ 2 - 3\ log.\ 10 + 3\ log.\ 2 = n.$$

$$10\ log.\ 2 = 3\ log.\ 10 + n = 3.010300$$

$$\therefore\ log.\ 2 = .3010300.$$

This method for calculating the logarithm of 2 is original. Hence, the *log.* of 5

$log.\ 10 = 1.000000$
$log.\ 2 = .301030$

$log.\ 5 = .698970$

7 *log.* 2 = *log.* 128 = 2.107210

3 *log.* 5 = *log.* 125 = 2.096910.

*log.* 125 = 2.096910

.003461 diff. found in example 1

*log.* 126 = 2.100371

.003433 diff. found in example 2.

*log.* 127 = 2.103804

.003406 diff. found in example 3.

*log.* 128 = 2.107210

The logarithms of 2, 200, 2000, &c., and of 5, 500, 5000, &c., also become known.

*Ques.* 5. Required the logarithms of 101, 2001, and 5001 ?

$$\frac{.868589}{201} = .004321$$

In this way the difference between the logarithms of 100 and of 101 is found,

$$\therefore \quad \textit{log.}\ 101 = 2.004321.$$

$$\frac{.868589}{4001} = .000217\ ;$$

but the logarithm of 2000 = 3.301030.

Therefore, *log.* 2001 = 3.301247.

$$\frac{.868589}{10001} = .000087$$

*log.* 5000 = 3.698970

3.699057 = *log.* 5001.

*Ques.* 6. The logarithm of 125 has been found = 2.096910, find the logarithms of 123, 124, 41, and 31 ?

$$2 \times 123 + 1 = 247.$$

$$\left.\begin{array}{l} \dfrac{.868589}{247} = .003517 \\ \dfrac{.868589}{249} = .003488 \end{array}\right\} \text{Differences required.}$$

$$\begin{array}{rl} \textit{log.}\ 125 = & 2.096910 \\ & .003488 \text{ difference.} \\ \hline \textit{log.}\ 124 = & 2.093422 \\ & .003517 \text{ difference.} \\ \hline \textit{log.}\ 123 = & 2.089905 \\ \hline \end{array}$$

$41 \times 3 = 123$

$31 \times 4 = 124$

$$\begin{array}{rl} \textit{Log.}\ 124 = & 2.093422 \\ \textit{log.}\ 4 = & .602060 \\ \hline \textit{Log.}\ 31 = & 1.491362 \\ \hline \end{array}$$

$2^{10} = 1024$

$$\textit{Log.}\ 1024 = 3.010300$$

$$\frac{.868589}{2049} = .000424$$

$$\textit{Log.}\ 1025 = 3.010724$$

$$\textit{Log.}\ 1025 = \textit{log.}\ (5 \times 5 \times 41) = 2\ \textit{log.}\ 5 + \textit{log.}\ 41$$

$$\therefore\ \textit{log.}\ 41 = \textit{log.}\ 1025 - 2\ \textit{log.}\ 5.$$

But the logarithms of 5 and 1025 are already found, consequently

$$\begin{aligned} \textit{Log.}\ 1025 &= 3.010724 \\ 2\ \textit{log.}\ 5 &= 1.397940 \\ \hline \textit{log.}\ 41 &= 1.612784 \end{aligned}$$

$$\begin{aligned} \textit{Log.}\ 123 &= 2.089905 \\ \textit{log.}\ 41 &= 2.612784 \\ \hline \textit{log.}\ 3 &= .477121 \end{aligned}$$

$$\begin{aligned} \textit{Log.}\ 126 = \textit{log.}\ (9 \times 2 \times 7) &= 2.100371 \\ \textit{log.}\ 9 &= .954242 \\ \hline & 1.146129 \\ \textit{log.}\ 2 &= .301030 \\ \hline \textit{log.}\ 7 &= .845098 \end{aligned}$$

In the same way may the logarithm of any given number be found.

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## MECHANICAL EXAMPLES.

### RULE I.

*If a force be exerted through a given space, and the power expended ascertained, then by quadrupling the force and doubling the space, it will be found that an expenditure of eight times the power will result ; this is true.*

RULE II.

*The resistance of a cylinder which moves in a fluid in the direction of its axis, is equal to the weight of a column of fluid, whose base is equal to that of the cylinder, and its altitude equal to the height through which a body must fall in vacuo by the force of gravity, to acquire the velocity of the moving body; this rule is also true.*

I will first find what resistance would be encountered by a plane of a foot square, urged through water at the rate of three miles an hour.

$$\frac{5280 \times 3}{60 \times 60} = 4.4 \text{ feet} = \text{space passed through in a second.}$$

In this case the well-known formula

$$\frac{v^2}{2g} \text{ gives } \frac{(4.4)^2}{2 \times 32\frac{1}{6}} = 3 \text{ ft. or } \frac{3}{10} \text{ of a foot, the height from}$$

which a body must fall to acquire a velocity of 4.4 feet a second.

$$62.5 \times .3 = 18.75 \text{ lbs}$$

This result may be tested by rigorous experiments made in France by D'Alembert, and other philosophers. They found that a surface of 1.1363 American feet with a velocity of 2.7263 feet per second, had a resistance equal to 8.234 lbs. avoirdupois, which is equal to 7¼ lbs. to the square foot.

Then if it be assumed, for it is near the truth, that resistances vary as the squares of the velocities,

$$(2.7263)^2 : 7\tfrac{1}{4} \text{ lbs.} :: (4.4)^2 : 18.8 \text{ lbs.}$$

almost exactly what is found by theory and Rule II.

Let us see how Beaufoy's experiments agree with theory:—the experiment was made on a foot square, the velocity was 9.2 miles an hour, and the resistance 203.79 lbs.

$$\therefore \quad (9.2)^2 : 203.79 \text{ lbs.} :: (3)^2 : 21.6 \text{ lbs.}$$

Theory makes the result 18.8 lbs.

If, for the sake of argument, we take 20 lbs. for the resistance, at a velocity of three miles an hour

$$3^2 : 20 \text{ lbs.} :: 6^2 : 80 \text{ lbs.}$$

which shows that 80 lbs. will be the resistance to a square foot at six miles an hour. The question in dispute is this; if the gravity of a weight of 20 lbs. is the measure of the resistance to a plane a foot square at the velocity of three miles an hour, and the gravity of a weight of 80 lbs. is the measure of resistance to the same plane at a velocity of six miles an hour, is the power necessarily expended per hour in the latter case eight-fold or four-fold, what it is in the former?

A quadrupled resistance is already predicated, which amounts purely from *doubled velocity*. Now, let the plane of a foot square be expanded to an area of 4 square feet, its resistance would then also be increased four-fold; but that would arise from *increased surface*. The error arises from considering these resistances as identical initial resistances, and then putting the term *pressure* as the representative of both. This is a fallacy. The one is absolute, the other *virtual*. Are they equal in effect? with regard to *time* they are. With regard to space they are not; but the cube theory requires that they should be with regard to space. To make this clear, imagine a pulley, the 20 lb. weight suspended over it by a cord attached to the one foot plane, a well being under the weight; also, imagine that the plane is in the water, at a convenient distance, that the cord weighs nothing, and that there is no rigidity or friction. Then the gravity of the weight pulls the plane through the water, and its velocity becomes constant when it moves at the rate of three miles an hour. Detach the 20 lb. weight and the one foot plane, and substitute the 80 lb. weight, and the plane of four feet. The velocity of this plane also becomes constant at three miles an hour, being an *actual* four-fold resistance, and a four-fold power

to sustain it. Take off the plane of four feet, attach the plane of one foot, and see what the 80 lb. weight will do with that. Its velocity becomes constant and uniform at the rate of six miles an hour. Time, that all important element, must now be introduced. The 20 lb. weight draws the plane of one foot three miles an hour. The 80 lb. weight draws the plane of four feet the same distance in the same time. Here, then, is an actual four-fold resistance, and a four-fold expenditure of power. But the plane of one foot can have but one-fourth the initial resistance of a plane of four feet. Therefore, to expend as much power upon the one foot plane as upon the plane of four feet, it must be drawn through the fluid double the distance. That is, it must be drawn through six miles instead of three in the same time; and the expenditure of power being four-fold for a doubled velocity, it is as the square of the velocity and not as the cube.

---

### FOR PADDLE-WHEEL STEAMERS.

Let $a$ = area of all the floats immersed, in square feet.

$b$ = the immersed angle of the paddle-wheels, at the centre of pressure.

$c$ = radius from the centre of motion to the centre of pressure of the floats.

$d$ = acting area in square feet.

$$d = a \cos. \frac{b}{3}$$

Suppose the area of all the immersed paddles $a = 294.6$ square feet, the angle $b = 47°$; what is the acting area?

$$\frac{47°}{3} = 15° \,..\, 40'\text{; Natural cos.} = 9628490$$

For practical purposes .963 will be sufficiently accurate.

$294.6 \times .963 = 283.6998$ square feet acting area.

Put $e$ = the length of the loadline of the vessel, in feet.

$f$ = the great immersed section, in square feet.

$g$ = area of resistance in square feet.

$h$ = displacement of the vessel in tons.

$i$ = horse power required to propel the vessel $j$ miles an hour, (statute miles.)

$k$ = co-efficient of the vessel.

$l$ = resistance of the vessel in pounds.

$m$ = tabular index corresponding to $k$.

$n$ = slip of either propeller or paddle-wheels.

$o$ = the pitch of the propeller.

$p$ = the number of revolutions a minute.

## EXAMPLES.

*Ques.* 1. Required the area of resistance of a vessel when $e = 260$ feet, = the length of the loadline; the greatest immersed section 500 square feet $= f$; the displacement of the vessel 2500 tons $= h$.

$$m = \frac{35\,h}{f\,e} = \frac{35 \times 2500}{500 \times 260} = .67$$

Opposite .67 in the following table, $k$, will be found = 1.76, the co-efficient of the vessel.

Then $g = \sqrt{\frac{f^3}{f + k\,e^2}} = \sqrt{\frac{(500)^3}{500 \times 1.76 + (260)^2}} = 32.$

square feet, the area of resistance required.

TABLE.

| $m$ | $k$ | $m$ | $k$ |
|---|---|---|---|
| .50 | 1.31 | .71 | 1.52 |
| .51 | 1.37 | .72 | 1.43 |
| .52 | 1.45 | .73 | 1.36 |
| .53 | 1.51 | .74 | 1.29 |
| .54 | 1.56 | .75 | 1.21 |
| .55 | 1.62 | .76 | 1.13 |
| .56 | 1.66 | .77 | 1.05 |
| .57 | 1.71 | .78 | .978 |
| .58 | 1.76 | .79 | .903 |
| .59 | 1.83 | .80 | .835 |
| .60 | 1.89 | .81 | .763 |
| .61 | 1.93 | .82 | .691 |
| .62 | 1.98 | .83 | .625 |
| .63 | 2.01 | .84 | .559 |
| .64 | 1.97 | .85 | .496 |
| .65 | 1.91 | .86 | .433 |
| .66 | 1.84 | .87 | .379 |
| .67 | 1.76 | .88 | .326 |
| .68 | 1.72 | .89 | .277 |
| .69 | 1.64 | .90 | .229 |
| .70 | 1.59 | .95 | .025 |

*Ques.* 2. Required the resistance of the vessel in pounds $= l$, when she is running at the rate of 10 miles an hour $= j$; the area of resistance $= 40$ feet $= g$, found as in the last example.

$$l = 4.22 \times g j^2 = 4.22 \times 40 \times 100 = 16880 \text{ lbs.},$$

the resistance of the vessel in pounds.

The resistance in pounds may be found by the following formula nearer the truth

$$Log.\ l = .633468 + log.\ g + 1.75\ log.\ j.$$

$$Log.\ j - log.\ 10 = 1.000000,$$

$$1.75\ log.\ j = 1.750000$$
$$log.\ g = 1.602060$$
$$\text{Constant} = .633468$$
$$Log.\ 9672.3 = 3.985528$$

The first rule gives the resistance = 16880 lbs

*Ques.* 3. Required the horse power $i$, when the speed is equal 11 knots an hour $= j$; and the area of resistance $= 33$ square feet $= g$.

$$i = \frac{j^3 \times g}{88} = \frac{11^3 \times 33}{88} = 49\tfrac{1}{8} \text{ horse power.}$$

The horse power may be found with greater accuracy by the succeeding logarithmic expression

$$Log.\ i = 2.75\ log.\ j + log.\ g - 1.944483$$
$$Log.\ j = 1.0413937\ ;\ log.\ g = 1.518514$$
$$2.75 \times 1.041393 = 2.863830$$
$$log.\ g = 1.518514$$
$$4.382344$$
$$\text{Constant} = 1.944483$$
$$Log.\ 274.07 = 2.437861$$

The horse power according to this latter method = 274.07 ; there is a discrepancy in these results also ; the cube of the speed is employed in one case, and $2\frac{3}{4}$ power in the other

$\frac{j^{2\frac{3}{4}} \times g}{88}$, put into a logarithmic form

gives the latter rule. Experiment has not yet settled this point in steam navigation, simple as it is, some employ the

square of the velocity, others the cube, and a third party the $2\frac{3}{4}$ power. Government experiments, both in this country and in England, are generally government jobs. Engineers are selected to superintend them upon the principal that determined who should be the village schoolmaster; one of those worthies, was chosen to fill the important office of village teacher, merely because he had a large family and a wooden leg.

### PROBLEM 4.

To find the slip of Propeller or Paddle-wheels, when the acting area, and area of resistance are given.

If the area of resistance = 49 feet, $= g$; and the acting area = 225 feet $= d$; required the slip $= n$.

$$n = \frac{\sqrt{g^3}}{\sqrt{g^3} + \sqrt{d^3}} = \frac{343}{343 + 3375} = \frac{343}{3718}.$$

### PROBLEM 5.

To calculate the power and find the properties of Paddle-wheels and Screw Propellers.

The pitch of a propeller = 33 feet; and makes 42 revolutions a minute, the slip = .35. What is the speed of the vessel?

$$\frac{33 \times 48}{88} (1 - .35) = 11.7 \text{ knots.}$$

Put $o$ = the pitch; $p$ = the revolutions a minute; and $n$ = the slip; then generally

$$j = \frac{o \times p}{88} (1 - n);$$

$$\therefore \quad p = \frac{88\, j}{o \times (1 - n)};$$

$$\therefore \quad n = 1 - \frac{88\, j}{o\, p}.$$

Let the speed be 11.7 knots the hour, $= j$; the slip $= .35 = n$; the pitch 33 feet $= o$; required the number of revolutions $= p$.

$$p = \frac{88\,j}{o \times (1 - n)} = \frac{88 \times 11.7}{33 \times (1 - .35} = 48$$

the number of revolutions a minute.

Required the slip, when the speed is 11.7 knots per hour, revolutions 48 per minute, and the pitch = 33 feet.

$$n = 1 - \frac{88\,j}{o \times p} = 1 - \frac{88 \times 11.7}{33 \times 48} = .35.$$

## PROBLEM 6.

ANY four of the five following quantities being given to find the fifth, namely, the radius of a paddle-wheel from the centre of motion to the centre of pressure of the floats, $= c$; the slip, $= n$; the immersed angle of the paddle-wheels at the centre of pressure $= b$; the statute miles per hour $= j$; and the revolutions a minute $= p$.

From the general expression

$$j = \frac{c\,p\,cos.\,\frac{1}{3}\,b}{14}\,(1 - n)$$

any one of the quantities is easily deduced, for

$$p = \frac{14\,j}{(1 - n)\,c\,cos.\,\frac{b}{3}}\;;\; n = \frac{14\,j}{p\,c\,cos.\,\frac{1}{3}\,b}.$$

Suppose the immersed angle of the paddle-wheels at the centre of pressure $= 54°..\,33' = b$. The slip $= .38 = n$; the revolutions per minute $= 16 = p$; and the radius of the paddle-wheel from the centre of motion to the centre of pressure of the floats $= 21$ feet; required the speed $= j$?

One third of $54° .. 33' = 18° .. 11'$, the natural cosine of which is $= .9500629$, for practical purposes .95 will be sufficiently near the truth.

$$j = \frac{c\,p\,\cos.\,\frac{1}{3}\,b}{14}\,(1 - n) = \frac{21 \times 16 \times .95}{14}\,(1 - .38) =$$

14.136 miles an hour.

Again, let the immersed angle = 54° .. 33′; the revolution per minute = 16; the speed 14.136 miles an hour; the radius of the paddle-wheel from the centre of motion to the centre of pressure = 21 feet; required the slip?

$$n = 1 - \frac{14 \times 14.136}{16 \times 21 \times .95} = 1 - .62 = .38$$

the required slip.

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## CALCULATIONS RESPECTING THE ANTI-SLIP SCREW PROPELLER.

Put $q$ = ¼ the pitch at the periphery = D E.

$r$ = ¼ the pitch at the hub = R S.

$s$ = the assumed slip in a fraction = H R, of $q$.

∴ $q = r + s$.

Put $t$ = extreme radius = Q D.

$u$ = diameter of the screw = $2t$ = F I.

$v$ = pitch of the propeller at the periphery = $4q$.

$w$ = angle of the blades at the periphery = K Q T.

$w_1$ = angle of the blades at the hub.

J M = $x$ = ¼ pitch of the propeller at the centre of effort of the blades.

B J = $y$ = .725 $t$, the radius at the centre of effort G, = O G = Q M.

∴ The actual pitch at the centre of effort = $4x$.

$z$ = horse power required to drive the propeller and ∝ revolutions a minute.

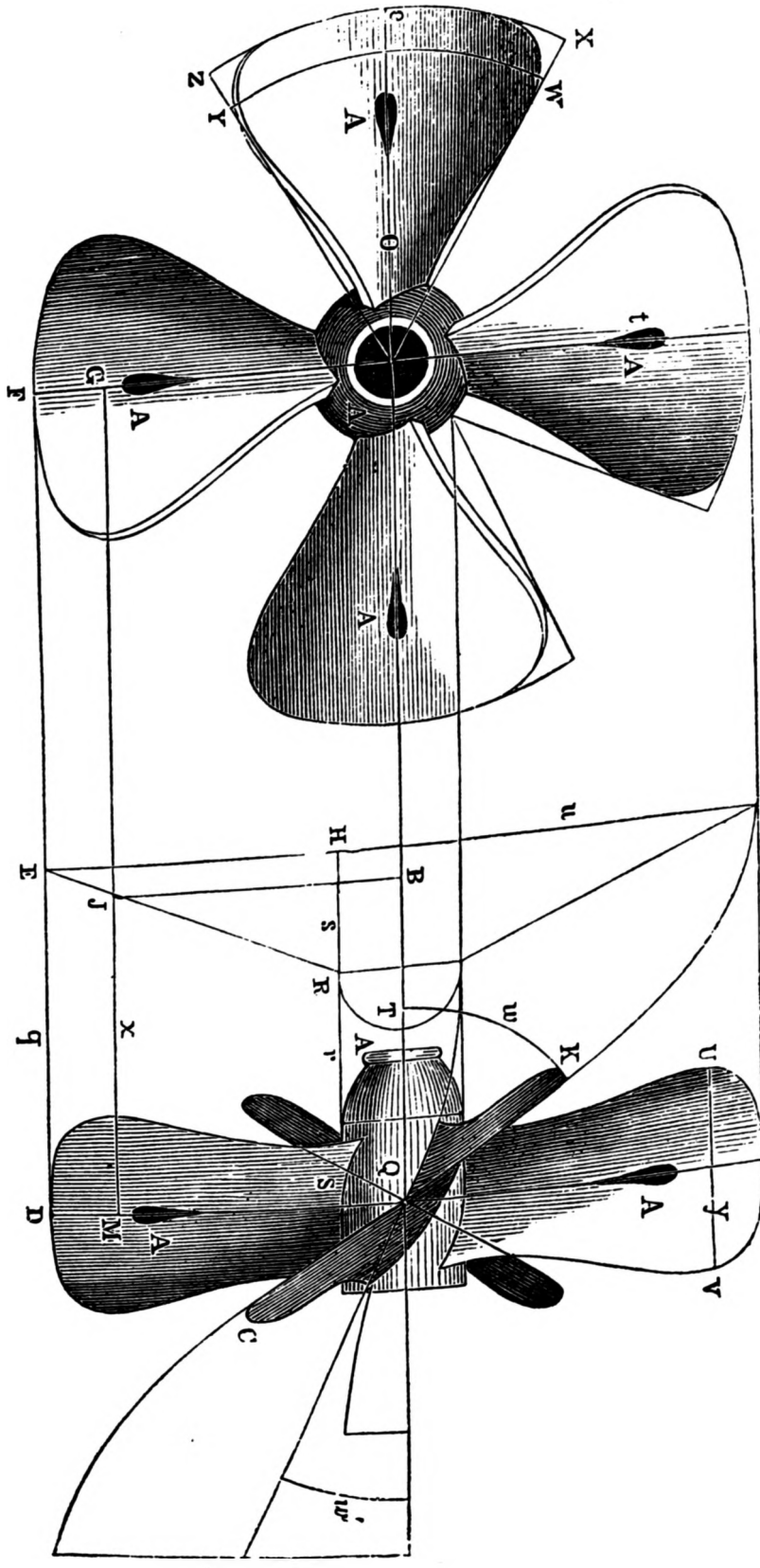


*Fig.* 17.

*Fig.* 16.

$\beta$ = number of blades.

$\gamma$ = length of blade parallel with the centre line = U V

$\delta$ = the breadth of the propeller blades over the edge between the corners X $z$

$\varepsilon$ = the circular arc in the angle $\theta$ = W O Y.

$\theta$ = the projected angle of the blades

A = the true inclined surface of the blades

A = the projected area of the blades.

$A_2$ = the acting area of the propeller.

Let $\pi$ = the circumference of a circle diameter = 1.

$Cot.\ w = \dfrac{v}{\pi u} = \dfrac{4q}{\pi 2t} = \dfrac{2q}{\pi t}$, the cotangent of the angle of the blades at the periphery.

$v = \pi u\ cot.\ w$, the pitch of the propeller at the periphery

$$v = \frac{360}{\theta} \times \gamma;$$

$$\theta = \frac{360}{v} \times \gamma;$$

$$v = \frac{\pi u \gamma}{\varepsilon}; \quad v = \frac{\pi u \gamma}{\sqrt{\delta^2 - \gamma^2}}; \quad \theta^2 = \frac{129600\,\gamma^2}{\quad}.$$

$\delta$ is the hypothenuse, and $\gamma$ is the base of a right angled plan triangle.

$\delta$ = K C measured over W Y, it is a projection the same as U V = $\gamma$.

$$\delta = \sqrt{\frac{\theta^2\,\pi^2\,w^2}{129600} - \gamma^2}$$

$$A_1 = \frac{.7854\,w^2\,\theta\,B}{360}$$

$$A = \frac{4\,t\,B\,(\delta + \gamma)}{9}.$$

$$A_2 = \frac{5\,u^7}{2\sqrt{v^2 + \pi^2 u^2}}$$

$$z = \frac{u^3 \propto^5}{480000}\,(\gamma\, s\, cos.\, w + \tfrac{1}{6})$$

---

PROBLEM 1.

THE pitch of the propeller = 33 feet; and makes .48 revolutions a minute, the slip = .35. What is the speed of the vessel?

$$\frac{33 \times 48}{88}\,(1 - .35) = 11.7 \text{ knot.}$$

By the introduction of the anti-slip screw, the slip would be reduced from 35 to 15 per cent., the speed will be

$$\frac{33 \times 48}{88}\,(1. - \cdot 15) = 15.3 \text{ knots.}$$

Put $o$ = the pitch;

$p$ = the revolutions a minute;

$n$ = the slip; and

$j$ = the knots the hour;

then generally

$$j = \frac{0 \times p}{88}\,(1 - n);$$

$$p = \frac{88\,j}{0 \times (1 - n)};$$

$$n = 1 - \frac{88\,j}{0 \times p};$$

$$o = \frac{88}{p\,(1 - n)}.$$

A ship fitted up with Byrne and Elliott's Screw made 13 knots an hour; the revolutions were 60 a minute, and the pitch 30 feet, what was the slip?

$$n = 1 - \frac{88\ j}{o \times p} = 1 - \frac{88 \times 18}{30 \times 60} = .12, \text{ or } 12 \text{ per cent.}$$

PROBLEM 2.

Given the diameter of a propeller = 12 feet, = $w$; and the angle at the periphery = 51° .. 33′ = $w$; to find the pitch = $v$?

*Cot.* 51° .. 33′ = $w$; .7940121

$$v = \pi\ u\ cot.\ w = 3.1416 \times 12 \times .794 = 29.93 \text{ feet.}$$

PROBLEM 3.

Given the diameter = 21 feet = $u$; the length of the blade parallel with the centre line = 6 feet = $\gamma$; the slip 40 per cent., or $s = .40$; the angle of the blades at the periphery = 63° .. 42′ = $w$; to find the horse power necessary to drive this screw 68 revolutions a minute = $\propto$.

Cosine 63° .. 42′ = .4430712

$$z = \frac{21^3 \times 68^3}{480000} (6 \times .4 \times .443 + \tfrac{1}{9}) = 7124.$$

| | |
|---|---|
| *Log.* 21 = | 1.32221 |
| *log.* 68 = | 1.832509 |
| | 3.154728 |
| | 3 |
| | 9.464184 |
| *log.* 480000 = | 5.681241 |
| | 3.782943 |
| *log.* of 1.1724 = | .069779 |
| *Log.* of 7124 = | 3.852722 |

As a second example suppose $u = 12$ feet ; $\gamma = 5$ feet ; $w = 57° .. 30'$ ; $s = 38$ per cent., or $s = 38$ ; what power is required to drive this propeller 40 revolutions a minute ?

$$z = \frac{15^3 \times 40^3}{480000} (5 \times .38 \times .5373 + \tfrac{1}{3}) = 509.4$$

| | |
|---|---|
| *Log.* 15 = | 1.176091 |
| *log.* 40 = | 1.602060 |
| | 2.778151 |
| | 3 |
| | 8.334453 |
| *log.* 1.13198 = | .053839 |
| | 8.388292 |
| *log.* 480000 = | 5.681241 |
| *log.* 509.4 = | 2.707051 |

## PROBLEM 4.

Given the diameter of a propeller = 21 feet = $u$ ; the angle of the blades at the periphery = $63° .. 42' = w$ ; the length $\gamma = 6$ feet ; the slip $s = .40$ or 40 per cent. ; four engines drive this screw of 7124 horse power = $z$ ; how many revolutions, $\propto$, will this propeller make in a minute ?

$$\propto = \frac{1}{u} \left\{ \frac{480000\, z}{(\gamma\, s\, cos.\, w + \tfrac{1}{3})} \right\}^{\frac{1}{3}} = 68 \text{ revolutions.}$$

$$\gamma\, s\, cos.\, w + \tfrac{1}{3} = 6 \times .4 \times .443 + \tfrac{1}{3} = 1.1743.$$

| | | |
|---|---|---|
| *Log.* 480000 | = | 5.681241 |
| *log.* 7124 | = | 3.852724 |
| | | 9.533965 |
| *log.* 1.1743 | = | .069779 |
| | | 3)9.464186 |
| | | 3.154729 |
| *log.* 21 | = | 1.322219 |
| *log.* 68. | = | 1.832510 |

PROBLEM. 5.

Given, the diameter of the propeller = 13 feet = $u$; the angle of the blades at the periphery = 59° .. 46′ = $w$; the angle of the blades at the hub = 11° .. 48′ = $w_1$; the diameter of the hub = 1.25 feet, = twice Q S; (see figure 16,) to find the pitch at the centre of effort.

Cotangent of 59° .. 46′ = .582793

Cotangent of 11° .. 48′ = 4.786730

The pitch of the periphery

$$= 13 \times .583 \times 3.1416 = 23.8 \text{ feet} = v$$

The pitch at the hub

$$= 1.25 \times 4.787 \times 3.1416 = 18.8 \text{ feet}, v_2$$

Put $v_1$ for the pitch at the centre of effort, $u_1$ the diameter at the centre of effort, and $u_2$ the diameter at the hub.

$$(v - v_2) : (v_1 - v_2) :: (u - u_2) : (.725\, u - u_2)$$

$$\therefore \quad v_1 = v_2 + \frac{(v - v_2)\ (.725\, u - w_2)}{u - u_2}$$

$$v_1 = 18.8 + \frac{5 \times 8.175}{11.75} = 22.3.$$

RECAPITULATION.

$v = 23 . 8$ $\quad u = 13$ feet

$v_1 = 22 . 3$ $\quad u_1 = .725\, u = 9.425$ feet

$v_2 = 18 . 8$ $\quad u_2 = 1.25$ feet.

## PROBLEM 6.

Given, the diameter of a propeller $= 13.25$ feet $= u$; and the angle $w = 65°$, to find the acting area of the propeller?

$$A_2 = \frac{5\, w^3}{2\sqrt{v^2 + \pi^2 w^2}};$$

but $v^2 = \pi^2 w^2 \cot.^2 w$, hence

$$A_2 = \frac{5\, u^3}{2\, \pi\, u\, \text{cosec. } w}.$$

| | | | |
|---|---|---|---|
| *log.* 13.5 = | 1.122216 | *log.* 2.5 = | .397940 |
| *log.* $\pi$ = | .497151 | 3 *log.* 13.5 = | 3.366648 |
| *log. cosec.* $w$ = | .042724 | | 3.764588 |
| | 1.662091 | | 1.662091 |
| | | *log.* 126.6 = | 2.102497 |

∴ Acting area of propeller = 126.6 square feet = $A_2$.

The ANTI-SLIP SCREW PROPELLER, Patented by the Author of this work, Mr. Oliver Byrne, in conjunction with J. G. Elliott, Esq. 5th October, 1858.

This propeller may be constructed of any of the known forms, and operated by any of the known methods of geering, but the form found to be the best, is that which has the centre of effort in the centre of the acting area of each blade; and when the water is discharged through the backs of the blades, the openings should be opposite, or symmetrically arranged to these centres. It has been found, that the power necessary to turn a solid metal screw and shaft, doing a given work, at a given velocity, needs not to be increased when a hollow shaft and screw through which water flows, are employed to do the same work, under the same circumstances.

Openings of different forms may be made in the blades, (see A, A, A, A, *Fig.* 16,) provided the action of the water passing through them, has no tendency to give a rotary motion of the screw.

A valve may be placed in any convenient place in the hollow shaft, or pipe, that conducts the water aft the propeller, to prevent its return in cases of back action.

*Fig.* 17, is an end view of a four blade screw propeller. A, A, A, A, are openings at the rear of the blades, opposite the centres of effort in the acting areas. A, A, A, A, communicate through passages with the hollow shaft, A Q, (*Fig.* 16.)

*Fig.* 16, is a longitudinal view of *Fig.* 17. The propeller is represented as permanently keyed on the shaft, but may be retained by a clutch.

As the propeller is moved, the advanced half of the blade O *e* X W, cuts into the water undisturbed by the action of the screw, and as the water passes O *e*, *Fig.* 17. the aft part of the blade ceases to be as efficient as the fore part O *e* W, on account of the thickness of the metal of which the propeller is formed; a supply of water through A, renders the aft part of the blade as effective in propelling the vessel as the far part. When water is conducted aft the propeller through a hollow shaft, or through openings in the blades, or through both shaft and blades at the same time, the slip of the screw is much diminished by the action of the blades against the water so discharged. Several contrivances may be employed to conduct the useless or superfluous water to the rear of the screw propeller, in immediate contact with the blades aft, not for the purpose of giving a rotary motion to the propeller, but for the purpose of diminishing what is termed slip.

THE END.

Nov. 3 1860.